

Geometry Literacy Scale (GLS): Measuring Pre-service Mathematics Teachers' Self-Perceived Competencies

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Abstract

Geometry is a fundamental component of mathematical proficiency, yet pre-service mathematics teachers (PMTs) often struggle to demonstrate the holistic literacy required to teach it effectively. Despite its importance, the literature lacks a measurement instrument to assess perceived geometry literacy among pre-service teachers. This study addresses this gap by defining the concept of “geometry literacy” and developing a valid and reliable Geometry Literacy Scale (GLS) tailored for PMTs. A rigorous scale development process was conducted with a large sample of 865 PMTs, split into two independent groups for Exploratory (EFA) and Confirmatory Factor Analysis (CFA). Analyses yielded a psychometrically sound 32 item instrument with a stable single-factor structure, reflecting the integrated nature of geometric thinking in adult learners. The scale demonstrated excellent internal consistency (Cronbach’s $\alpha = .94$) and composite reliability (CR = .95). CFA results confirmed that the majority model fit indices indicated acceptable fit. The GLS provides researchers and teacher educators with a valid and reliable instrument for assessing PMTs’ self-perceived geometry literacy and may support future research, program evaluation, and the improvement of geometry teacher education.

Key words: Geometry literacy, Mathematics education, Pre-service teachers, Scale development, Structural validity

Introduction

“Let no one ignorant of geometry enter!”

Platon (BC 428/427 or 424/423-348/347)

Geometry encompasses individuals’ ability to analyze geometric shapes and structures, understand the properties and relationships among them, construct mathematical arguments regarding geometric relationships, and employ geometric modeling, spatial reasoning, and visualization in problem-solving (National Council of Teachers of Mathematics [NCTM], 2000). It further contributes to development of skills such as comparison, generalization, and summarization (Bal, 2014). Beyond classroom mathematics, geometry finds application in

fields such as architecture, art, and urban planning; its connections to real-life situations offer students richer opportunities for meaningful understanding compared to other mathematical content (Hardianti et al., 2017). In essence, geometry aims to facilitate comprehension of properties of geometric objects in two- and three-dimensional spaces, formalize geometric space, express transformations, and prove propositions (Baki, 2006).

Although geometry is an important component of the mathematics curriculum, it is perceived as a complex part and is thought to have no connection with daily life (Patkin & Levenberg, 2012). Not being literate in geometry could be a reason for why this connection was not established. When we focus on the literature about geometry, we see that there are some conclusions that could be thought as negative. For example, Kılıç et al. (2015) determined that 41% of the pre-service mathematics teachers (PMTs) who participated in their study had misconceptions about the concept of angle. Similarly, it was revealed that 40.10% of the candidates had misconceptions about the concept of right angle. It was observed that half of the candidates (51.70%) gave incomplete or incorrect answers regarding the concept of line segment. As a matter of fact, they stated that the basis of these misconceptions stems from the lack of knowledge about concept definitions and properties. These results indicate that pre-service teachers (PSTs) do not have sufficient knowledge of geometry concepts. It can be said that this is related to whether they are geometry literate or not. Because just as an individual who is mathematically literate knows the definitions of mathematical concepts, an individual who is literate in geometry will also know the definitions of geometry concepts. Kılıç et al. (2015) also suggested that geometry teaching should be organized toward the correct understanding of the concepts. In summary, attention should be paid to being literate in geometry.

In addition to these, studies show that PSTs' geometric thinking levels are not at the required level. For example, Akay and Kurtuluş (2017) concluded that PMTs were concentrated at the informal deduction level (level 2), while PSTs who had students in other departments were concentrated at level 1. Akin and Sezgin (2022) likewise found that students from both the elementary mathematics teaching ($n = 89$) and classroom teaching ($n = 113$) departments demonstrated performance corresponding to levels 1 and 2. Osmanoğlu (2019) concluded that the PSTs were mainly at the informal deduction level. He also found that there were

deficiencies such as having incomplete knowledge about the properties of shapes and establishing incorrect relationships between shapes. This indicates that one reason for the result is geometry literacy. Therefore, being able to explain geometric concepts, use different representations, explain the relationships between shapes and objects, solve geometry problems, perform modeling, and recognize geometry in daily life can be considered as the characteristics that an individual who is literate in geometry should have. To accurately identify and assess these competencies, valid and reliable measurement tools are necessary.

The existing literature presents numerous studies focused on the development of scales. For example, Bekdemir and Duran (2012) developed a Visual Mathematics Literacy (VML) self-efficacy perception scale targeting primary school students. Their initial 58 item scale, structured on a five-point Likert format, was administered to 428 students. Following factor analysis, items with low loadings were removed, yielding a final 38 item scale organized into three factors: Field Content, Process, and Place of Use. These factors, aligned with the Program for International Student Assessment (PISA) definition of mathematical literacy, explained 41.81% of the total variance, and the scale demonstrated high reliability with a Cronbach's alpha (α) of .943. Similarly, İlhan (2015) developed a VML scale for PMTs. Starting with a 60-item pool based on literature review and expert input, a 54-item draft was administered to 325 PMTs. Subsequent factor analyses reduced the scale to 37 items across five sub-factors: Visual Perception, Geometric Domain, Spatial Intelligence, Concrete Representation, and Pattern. The scale showed strong reliability ($\alpha = .904$) and accounted for 44.25% of the total variance. This scale was further employed to examine the relationship between PMTs' VML levels and their geometry achievement. Baypınar (2017) designed a 30-item scale to assess seventh-grade students' perceptions of mathematical literacy. Utilizing a five-point Likert format, the scale development process involved six stages and incorporated both exploratory (EFA) and confirmatory (CFA) factor analyses. The resulting four-factor structure -Mathematical Skills, Personal Experience, Scientific Modeling, and Social Context- explained 48.34% of the variance, with confirmatory fit indices exceeding acceptable thresholds. The scale's reliability was high, with a Cronbach's α of .925. Katrancı and Şengül (2019) developed a mathematical literacy scale for middle school students, starting from an initial 49-item pool. EFA supported a unidimensional structure, retaining 34 items that accounted for 47.54% of the total variance. CFA confirmed the scale's excellent fit, and

reliability measures indicated sufficient internal consistency. Despite the availability of valid and reliable instruments, there is a notable gap in the literature regarding scales specifically designed to assess the geometry literacy of PMTs. Therefore, the present study aims to develop a geometry literacy scale tailored for PMTs. While mathematical literacy is often assessed through cognitive performance tests, researchers such as Bekdemir and Duran (2012) and İlhan (2015) have approached literacy constructs through the lens of self-efficacy perceptions. They argue that individuals' beliefs about their capabilities are strong predictors of their actual behavior. Aligning with this theoretical perspective, the GLS was constructed not as a cognitive achievement test, but as a self-report instrument designed to measure PMTs' self-perceived competencies regarding geometry literacy. It is important to clarify that the GLS does not directly measure observable geometry performance. Rather, it assesses PSTs' self-perceived competencies related to geometry literacy. In line with self-efficacy theory (Bandura, 1997), such perceptions are considered meaningful predictors of future instructional behavior and professional practice. Therefore, the construct measured in this study can be more precisely described as geometry literacy self-efficacy perception, reflecting candidates' beliefs about their capacity to enact geometry literacy in educational contexts.

In conclusion, this study offers several significant contributions. Geometry, as a fundamental component of mathematics education, plays a critical role in the development of students' spatial reasoning, problem-solving, and critical thinking skills. However, research indicates that PMTs often possess common misconceptions about basic geometric concepts and demonstrate inadequate levels of geometric thinking. These deficiencies hinder their ability to teach geometry effectively and contribute to the perception of geometry as an abstract discipline disconnected from everyday life. One of the main underlying causes of this issue is a lack of geometry literacy, which involves not only conceptual understanding but also the ability to apply geometric reasoning in various contexts. Despite its importance, geometry literacy has not been explicitly conceptualized and operationalized as a distinct construct in the literature. Thus, the first contribution of this study is to offer a conceptual framework and a clear definition of geometry literacy, thereby addressing a critical gap in the field. Moreover, there is no existing valid and reliable measurement tool specifically developed to assess the geometry literacy of PMTs. Therefore, the second key contribution of this study is the development of a scientifically sound instrument designed to evaluate geometry literacy in

this target group. The use of this scale is expected to provide meaningful insights into PSTs' competencies and support the improvement of teacher education programs. In the long term, this will contribute to enhancing the quality of geometry instruction in schools. To our knowledge, this study represents one of the first attempts to systematically conceptualize geometry literacy and to develop and validate an instrument specifically designed to assess this construct for PMTs.

In the remainder of the study, we review the literature and provide a definition of geometry literacy along with its key characteristics. Subsequently, we developed a scale to measure geometry literacy by following the steps proposed by Hinkin (1998), which include item generation, content validation, scale administration, item reduction, CFA, and reliability testing.

Conceptual Framework

PISA is research conducted by the Organization for Economic Co-operation and Development (OECD) that evaluates the knowledge and skills acquired by students in the 15-year-old group over three years. Its main purpose is to measure students' ability to use the knowledge and skills they learn at school in daily life. Evaluations were conducted in the areas of reading skills, science literacy, and mathematical literacy. In this context, literacy is expressed as finding, using, accepting, and evaluating written resources to enable students to participate and contribute to society more effectively by improving their knowledge and potential (Ministry of National Education [MoNE], 2019). Within the framework of this context, the concepts of mathematical literacy, visual literacy, and VML emerge. Mathematical literacy refers to an individual's ability to formulate, apply, and interpret mathematics in various contexts. In this definition, emphasis is placed on developing students' capacity to use mathematics in daily contexts (OECD, 2019). To have mathematical literacy skills, it is necessary to gain skills and competencies at various levels such as mathematical thinking and comprehension, performing mathematical operations, having knowledge of mathematical content, and making applications (Durasi et al., 2019). The characteristics of a mathematically literate individual are discussed in four dimensions which are as follows:

- Mathematics content,

- Mathematical process (thinking),
- Historical development of mathematics,
- Actuality (Tekin & Tekin, 2004).

In addition, visual literacy is defined as the ability to read, understand, and interpret information presented pictorially and graphically (Wileman, 1993) and the ability to analyze, interpret, and create visual messages by reading them correctly (Yeh, 2008). Visual literacy is an important skill for mathematics, which is a branch of science in which drawings, charts, graphs, and visuals are used extensively (Ev-Çimen & Aygüner, 2018). The concept of VML has emerged because of the use of drawings, tables, and geometric shapes in almost all mathematics subjects (Katrancı & Şengül, 2019). VML is defined as evaluating daily life problems visually and visual information mathematically (Bekdemir & Duran, 2012).

The first specific purpose of our country's mathematics curriculum in the context of students is to develop their mathematical literacy skills. When the program is examined, mathematics course at primary school level; while it consists of four learning areas: numbers and operations, geometry, measurement, and data processing, at the middle school level it consists of five learning areas: numbers and operations, algebra, geometry and measurement, data processing, and probability (MoNE, 2018). It is important to develop literacy skills in these sub-learning areas to fully acquire mathematical literacy skills. The focus should be on the concept of visual literacy and VML as well as geometry literacy.

Furthermore, VML is expressed as the process of using paper-pencil, technology, and mental activities while structuring the images encountered and using these images for mathematical understanding (Çilingir-Altınır & Dinç-Artut, 2017). It was determined that the VML self-efficacy perceptions of middle school students (Şengül et al., 2017) were at a medium level, and that of PMTs was above the intermediate level (Topbaş-Tat, 2018). There are also studies showing that middle school students' perceptions of VML self-efficacy are high (Çelik, 2019; Katrancı & Şengül, 2019). Tutkun et al. (2014) determined that middle school students' perception of VML is high, and as their level of mathematics achievement increases, their perception of VML also increases. In their study, İlhan and Aslaner (2021) concluded that PMTs' VML perceptions had a positive effect on their geometry performance. Within the

framework of the definitions given above, geometry literacy can be defined as an individual's capacity to formulate, use, and interpret geometry. Additionally, geometry literacy can be interpreted as an individual's capacity to recognize and understand the role geometry plays in the world around him/her by using geometric thinking and decision-making processes in solving the problems he/she will encounter. According to these, Figure 1 shows the characteristics that an individual who is literate in geometry should have.

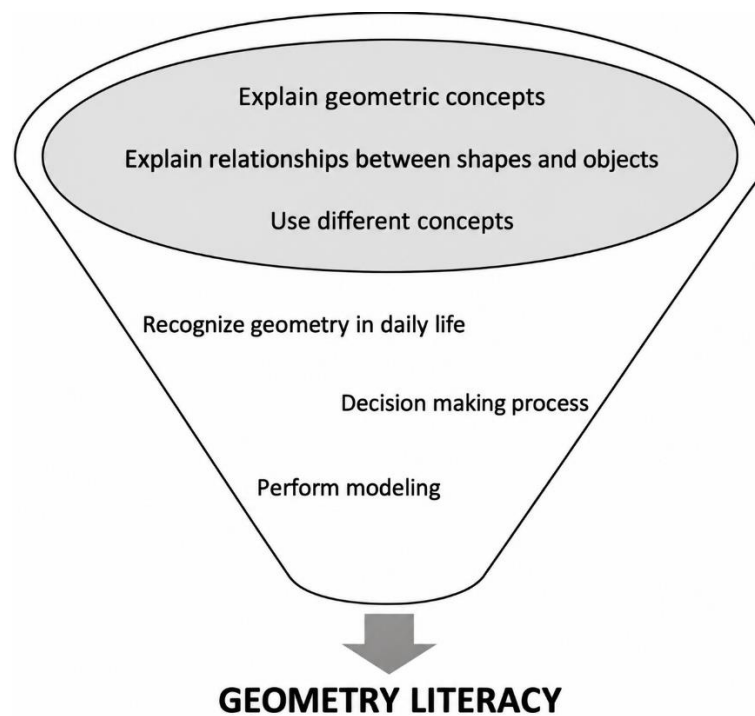


Figure 1. The Characteristics Indicative of Geometry Literacy

Figure 1 illustrates the conceptual framework of geometry literacy proposed in this study. Visualized as a funnel, the diagram suggests that geometry literacy is not merely a collection of isolated skills, but rather a synthesis of various cognitive processes. These processes range from theoretical competencies, such as explaining geometric concepts and relationships, to practical applications like performing modeling and decision-making in daily life contexts. The funnel metaphor emphasizes that these distinct components converge and integrate to form the holistic competency defined as “Geometry Literacy” for PMTs.

Method

The study was designed according to a quantitative approach to develop a scale of geometry literacy for PMTs. To achieve this, we followed the procedural steps outlined by Hinkin (1998), including item generation, content validation, scale administration, item reduction, CFA, and reliability testing.

Item Generation

Because geometry literacy has not been explicitly operationalized as a distinct construct in the literature, a comprehensive review was conducted to establish a conceptual foundation. A conceptual definition of geometry literacy was made based on the concepts of mathematical literacy (OECD, 2019), visual literacy (Wileman, 1993; Yeh, 2008), and VML (Bekdemir & Duran, 2012). Therefore, geometry literacy refers to an individual's ability to conceptualize, apply, and make sense of geometric principles. It encompasses not only the use of geometric knowledge in problem-solving but also the capacity to perceive and interpret the presence and function of geometry in real-world contexts through geometric reasoning and informed decision-making.

Subsequently, scale development studies on mathematical literacy and VML (Acar, 2016; Baypınar, 2017; Bekdemir & Duran, 2012; İlhan, 2015; Katrancı & Şengül, 2019; Kükey, 2013; Özgen & Bindak, 2008) were examined. Finally, an item pool of 40 items was prepared, considering the conceptual definition of geometry literacy, the scales mentioned above, and our country's mathematics curriculum (as the PMTs participating in the study will teach at this level when they graduate) (MoNE, 2018).

Given that the conceptual framework draws upon self-efficacy theory, the items were structured to elicit PMTs' beliefs about their capabilities rather than measuring their direct cognitive knowledge. Therefore, items were formulated using "I can..." statements (e.g., "I can use my geometry knowledge in my daily life") to assess their geometry literacy self-efficacy, consistent with the methodology employed by Bekdemir and Duran (2012).

Content Validation

Experts were asked to evaluate the items in terms of scope and content. An expert opinion form (EOF) had been created. A short example from the EOF is as follows:

Table 1.

An Example of an EOF

No	Draft Items	Suitable	Unsuitable	General evaluation
1	I know the basic symbols in geometry and their meanings (perpendicularity, parallelism, etc.).			
2	I can say which geometric shape it is, given its necessary and sufficient defining properties.			
Suggested item(s):				

Moreover, a general evaluation is expected in terms of spelling rules, grammar, lack of expression, and comprehensibility. In addition, if there are any suggested item(s), they should be written down. This form was sent to seven mathematics educators and five mathematics teachers. Because content validity assessment is primarily a cognitive task that does not necessitate detailed knowledge of the construct itself, the use of a small sample was considered appropriate (Anderson & Gerbing, 1991; Hinkin, 1998; Schriesheim et al., 1993). Feedback from experts and mathematics teachers was examined, and necessary corrections were made. For example, the expression *"I can write the definition of circle"* had been corrected as *"I can define the concept of circle"*. The statement *"I can use what I learned in geometry in my daily life"* had been corrected as *"I can use my geometry knowledge in my daily life"*. The statement *"I can estimate the area of a field using my knowledge of geometry"* was removed because it was similar to another item. The suggested items were also examined, and items that were thought to not compromise content validity were added.

Scale Administration

A draft version of the 42-item scale was created. 21 of these items contain positive statements and 21 contain negative statements. Finally, these items were answered in a 5-point Likert type (strongly disagree, disagree, neither agree nor disagree, agree, and completely agree). A

link was created to answer the scale, and this link was shared with PMTs studying in the Faculties of Education in Türkiye. The approximate response time of the scale is 10-15 minutes. Consequently, two sets of samples were collected: Sample A for item reduction and Sample B for model validation and confirmation. The detailed information of the PMTs is presented in Table 2.

Table 2.

The Information of PMTs

	Sample A			Sample B		
	Female (F)	Male (M)	Total (T)	F	M	T
Freshmen	95	28	123	94	29	123
Sophomores	73	31	104	85	18	103
Juniors	88	29	117	79	39	118
Seniors	66	22	88	61	28	89
T	322 (74.54%)	110 (25.46%)	432	319 (73.67%)	114 (26.33%)	433

In scale development studies, the adequacy of the sample size for factor analysis is often evaluated using various benchmarks. According to Comrey and Lee (1992), a sample size of 50 is considered poor, 300 is good, and 1000 is excellent. In contrast, Kline (1994) suggests that a sample size of 200 is sufficient. Additionally, it has been recommended that the number of participants should be at least ten times the number of items in the scale to ensure the robustness of the factor structure. The draft version of the scale consists of 42 items so that a sample of 420 people would be enough. There were 432 PMTs in Sample A and 433 PMTs in Sample B. In total 865 PMTs participated in this study. Therefore, the samples were sufficient for analysis.

Analyses and Results

Item Reduction

The suitability of the collected data for factor analysis is determined by sampling the adequacy values. In addition, the prerequisite for performing factor analysis is a certain degree of

correlation between the variables. While the Kaiser-Meyer-Olkin (KMO) result is the sampling adequacy statistic, Bartlett's test of sphericity (BTS) shows whether there is a sufficient relationship between variables (Sipahi et al., 2010). In this context, these test results were first evaluated. Table 3 shows the findings of these test results.

Table 3.

Findings of the KMO and BTS

KMO measure of sampling adequacy		.94
BTS	Approx. Chi-Square	7781.34
	df	861
	Sig.	.00

If the KMO value is .80 and above, the variables are at an excellent level for factor analysis, and if the p -value of the Bartlett test is less than .05 significance level, there is a sufficient level of relationship between the variables for factor analysis (Sipahi et al., 2010). According to Table 3, the KMO value is .94, which is at an excellent level, and the significance value of BTS is less than .05. These findings demonstrate the suitability of the data for factor analysis. At this point, the suitability of the items for factor analysis should be checked one by one. This is controlled by measuring the sampling rate (MSR), and if the results are .80 and above, the results are interpreted as excellent. If a result below .50 is obtained, this item should be removed from the analysis (Sipahi et al., 2010). Accordingly, MSR values for all items were calculated, and the results were determined to vary between .84 and .98. Thus, it was decided that all items were suitable for factor analysis, and the analysis was carried out.

To determine the number of factors, the eigenvalue statistics, scree plot, and explained variance ratio by the factors are considered (Büyüköztürk, 2020). According to the eigenvalue statistics, there should be eight factors, and according to the scree plot, there should be one factor. When the variance rates of the eight factors were examined, the variance rate of the first factor was 31.99%. In single-factor scales, a ratio of 30% or higher is considered sufficient (Büyüköztürk, 2020). Accordingly, it was decided that the number of factors should be one. A scree plot of the scale is shown in Figure 2.

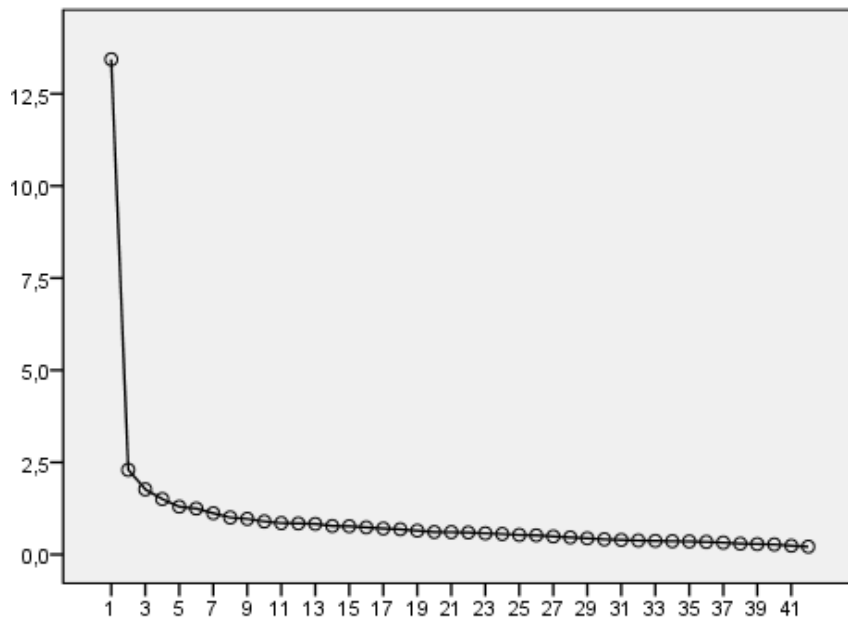


Figure 2. Scree plot of the GLS

Figure 2 illustrates the scree plot derived from the EFA, plotting the eigenvalues against the component numbers. As clearly observed in the graph, there is a sharp decline (break point) after the first factor, followed by a horizontal plateau. This distinct “elbow” shape provides visual evidence supporting the unidimensional structure, indicating that the contribution of subsequent factors to the total variance is negligible compared to the first factor.

In addition, factor loading values should be considered. Although there is no agreed-upon criterion, some researchers eliminated items with factor loads below .50, whereas others set this limit as .70 (Sipahi et al., 2010). In this study, we decided to remove items with factor loading values below .50. Accordingly, a structure consisting of 32-items and a single-factor was revealed. The findings of the EFA are presented in Table 4.

Table 4.

Findings of the EFA of the GLS

Item no	Item	Factor loading	Common variance
gl23	I can explain the relationships between geometric solids.	.68	.46
gl22*	I cannot solve problems related to geometric solids.	.66	.44

gl19*	I cannot model a geometric solid whose definition is given.	.66	.44
gl26	I can solve a geometry problem by using definitions.	.65	.42
gl35	I can use my geometry knowledge to make sense of social and current events.	.65	.43
gl20	I can create relations regarding the surface area measurements of the geometric solid.	.65	.42
gl36*	I do not notice geometric relationships in scientific events.	.65	.42
gl32	I can use my geometry knowledge in my daily life.	.65	.42
gl8*	I cannot explain the relationships between the geometric shapes.	.64	.41
gl4	I can explain the relationships between geometric concepts.	.64	.41
gl24	I can summarize a geometry problem using different representations, such as tables, graphs, and symbols.	.64	.41
gl10*	I cannot form the area relations of the geometric shapes.	.63	.40
gl40	I can estimate the length, area, and volume using similarity in daily life.	.63	.39
gl17*	I cannot decide which geometric object the given different development belongs to.	.63	.39
gl42	I can create geometric shapes and solids using concrete models.	.63	.39
gl21*	I cannot create relations regarding the volume measurements of the geometric solid.	.62	.39
gl18*	I cannot explain the properties of the geometric objects.	.62	.39
gl38	I can estimate the area of any place (e.g. classroom, living room, etc.) using my geometry knowledge.	.62	.39
gl30*	I cannot use my geometry knowledge to solve the problems I encounter in daily life.	.60	.36
gl25*	I cannot solve a geometry problem in different ways using representations such as graphical, algebraic, and numerical.	.60	.36
gl34*	I cannot relate social and current events to my geometry knowledge.	.59	.35
gl16	I can easily name geometric solids (e.g. prism, cylinder, etc.) given their distinctive features.	.59	.35
gl27*	I cannot solve a geometry problem using concrete materials (e.g. geometry board, compass, quadrant, etc.)	.58	.34
gl13	I can solve problems related to geometric shapes.	.58	.33
gl37	I can use my geometry knowledge to describe an address.	.56	.32

gl41*	I cannot notice geometric shapes found in nature (e.g. snowflake, snail, etc.).	.54	.29
gl29*	I cannot properly use checkered, dotted, or isometric paper when solving a geometry problem.	.53	.28
gl15	I can prove in different ways that the sum of the interior angles of a triangle is 180°.	.53	.28
gl31	I can exemplify geometric shapes and solids in daily life.	.52	.27
gl33*	My geometry knowledge does not help me in my other courses.	.52	.27
gl5	I can show a particular angle with different representations.	.51	.26
gl7	I can define the concept of the circle.	.51	.26
The total explained variance = 36.60%			

* Items should be coded in reverse.

Table 4 presents the detailed results of the EFA, including factor loadings and common variances for the final 32 items. As seen in the table, all retained items exhibited factor loadings above the determined threshold of .50, confirming the strength of the items' relationship with the construct. These loadings range from a minimum of .51 (Items gl5 and gl7) to a maximum of .68 (Item gl23). Additionally, items marked with an asterisk (*) represent negatively worded statements, which were reverse coded prior to analysis. The analysis yielded a single-factor structure that accounts for 36.60% of the total variance, with common variance values ranging between .26 and .46.

Model Validation

The structure consisting of 32 items and a single-factor because of EFA was tested via CFA. First, the *t*-values were investigated to exclude non-significant *t*-values from the analysis (Çokluk et al., 2010). Table 5 shows the *t*-values, standardized factor loadings, and R^2 of the GLS.

Table 5.

CFA Findings of the GLS

Items no	t-values	Standardized factor loadings	R ²
gl4	13.53	.64	.41
gl5	11.56	.72	.52
gl7	10.22	.78	.61
gl8	14.47	.60	.36
gl10	13.45	.64	.41
gl13	15.04	.57	.32
gl15	9.29	.81	.66
gl16	14.73	.59	.35
gl17	14.77	.58	.34
gl18	16.90	.49	.24
gl19	16.65	.50	.25
gl20	14.37	.60	.36
gl21	15.96	.53	.28
gl22	17.40	.47	.22
gl23	16.34	.51	.26
gl24	14.41	.60	.36
gl25	13.48	.64	.41
gl26	14.76	.58	.34
gl27	14.87	.58	.34
gl29	13.92	.62	.38
gl30	15.83	.54	.29
gl31	13.46	.64	.41
gl32	16.49	.51	.26
gl33	12.95	.66	.44
gl34	12.36	.69	.48
gl35	12.33	.69	.48

gl36	14.52	.60	.36
gl37	8.78	.83	.69
gl38	12.18	.70	.49
gl40	12.30	.69	.48
gl41	12.20	.70	.49
gl42	14.41	.60	.36

Table 5 displays the parameter estimates resulting from the CFA, including t -values, standardized factor loadings, and squared multiple correlations (R^2). As indicated in the table, all t -values were found to be statistically significant ($p < .01$), ranging from 8.78 (Item gl37) to 17.40 (Item gl22), which confirms that all items significantly load onto the latent geometry literacy factor. The standardized factor loadings varied between .47 (Item gl22) and .83 (Item gl37). While the majority of loadings exceeded the conventional .50 threshold, two items (gl18 and gl22) showed values slightly below this cut-off (.49 and .47, respectively) but were retained due to their significant t -values and theoretical contribution to the construct. Furthermore, the R^2 values, indicating the amount of variance explained by each item, ranged from .22 to .69. The other results of CFA are shown in Table 6.

Table 6.

Goodness of Fit Indices of the CFA of GLS

Indices	Result	Moderate level/Good fit	Perfect fit	References
$\frac{X^2}{df}$	5.78	≤ 5	≤ 3	(Kline, 2005; Sümer, 2000)
RMSEA	.10	$\leq .07$	$\leq .05$	(Brown, 2006; Steiger, 2007; Tabachnick & Fidell, 2001)
RMR	.06	$\leq .08$	$\leq .05$	(Brown, 2006; Hu & Bentler, 1999)
SRMR	.06	$\leq .08$	$\leq .05$	(Brown, 2006; Hu & Bentler, 1999)
NFI	.94	$\geq .90$	$\geq .95$	(Hu & Bentler, 1999; Sümer, 2000; Thompson, 2004)
NNFI	.95	$\geq .90$	$\geq .95$	(Hu & Bentler, 1999; Sümer, 2000; Thompson, 2004)

CFI	.95	$\geq .90$	$\geq .95$	(Hu & Bentler, 1999; Sümer, 2000; Thompson, 2004)
IFI	.95	$.90 \leq IFI \leq .95$	$.95 \leq IFI \leq 1.00$	(quoted from İlhan & Çetin, 2014)
RFI	.93	$.90 \leq RFI \leq .95$	$.95 \leq RFI \leq 1.00$	(quoted from İlhan & Çetin, 2014)
PNFI	.87	$.50 \leq PNFI \leq .95$	$.95 \leq PNFI \leq 1.00$	(Hu & Bentler, 1999)

Although the RMSEA value (.10) exceeds recommended cutoffs for close model fit (Browne & Cudeck, 1993), model evaluation should not rely solely on a single cutoff criterion (Hu & Bentler, 1999; Marsh et al., 2004). Incremental fit indices (CFI = .95; IFI = .95; NFI = .94) indicate strong comparative fit. Furthermore, prior research suggests that RMSEA may be influenced by model specification and structural characteristics (Kenny et al., 2015). Considering the overall pattern of fit statistics, the single-factor model was retained as an acceptable representation of the data. Furthermore, an examination of the measurement model indicated that error variances ranged between .47 and .81, and no item with disproportionately high error variance was detected.

In the refinement process, modification indices were also examined to identify potential error covariances that could artificially lower the RMSEA value. Although defining covariances between error terms of items with similar wording could improve fit statistics, no modifications were made because there was no strong theoretical justification for linking specific item errors. We prioritized model parsimony over artificially inflating fit indices. Furthermore, as Kenny et al. (2015) argue, RMSEA can be problematic and upwardly biased in models with small degrees of freedom or specific sample structures. Given that the CFI, NFI, and IFI values (all $\geq .94$) indicate an excellent fit, the elevated RMSEA is interpreted as an artifact of the model's complexity rather than a sign of misspecification.

Reliability and Validity

As a measure of internal consistency, Cronbach's α is considered acceptable when it exceeds .70 (DeVellis & Thorpe, 2021). Composite reliability (CR), derived from structural equation modeling, is conceptually equivalent to coefficient omega (Padilla & Divers, 2016). Bagozzi (1981) defined convergent validity as the internal consistency among indicators measuring the same construct; therefore, researchers have used reliability estimates as one of the criteria

for assessing convergent validity (Fornell & Larcker, 1981). A CR value of at least .70 is commonly used as a threshold for acceptable internal consistency (Hair et al., 2009). The reliability of GLS was determined using Cronbach's α as well as CR that was used to assess the scale's convergent validity. The value of Cronbach's α for Sample A was .94, whereas the Cronbach's α value for Sample B was .94 as well as CR for Sample B was .95. The average variance extracted (AVE) for the single-factor structure was calculated as .40. Although this value falls below the conventional .50 threshold suggested by Fornell and Larcker (1981), the composite reliability exceeded .70, indicating acceptable internal consistency despite moderate shared variance among the indicators.

Discussion

In this study, a valid and reliable measurement tool was developed to assess PMTs' perceived geometry literacy. The EFA results indicated that the single-factor structure accounts for 36.60% of the total variance. While some psychometric guidelines suggest explained variance rates of 50% or higher, interpreting this value requires consideration of the construct's breadth and the instrument's length. As emphasized by Peterson (2000), explained variance in factor analysis is inversely related to the number of items; scales with a larger number of items ($n > 30$) typically yield lower explained variance compared to shorter scales. Given that the GLS consists of 32 items designed to cover the comprehensive "funnel" framework (Figure 1) – ranging from definitions to complex modeling and daily life applications – a moderate variance is an expected outcome of high content validity. Reducing the item pool solely to maximize statistical variance would have narrowed the construct's scope, failing to capture the holistic nature of "literacy" (Henson & Roberts, 2006). Therefore, priority was placed on preserving the ecological validity and representativeness of the construct rather than maximizing variance at the expense of theoretical depth. Furthermore, the high internal consistency (Cronbach's $\alpha = .94$) confirms that, despite the moderate variance explained, the items function coherently to measure a unified construct. Accordingly, given the large sample size ($n = 865$) and the extensive item pool, the obtained variance aligns with expectations for comprehensive single-factor instruments in educational research.

It may be argued that adopting a multidimensional structure – separating components such as visual perception, modeling, and real-life applications – could have increased the total explained variance. However, within the conceptual framework of this study, geometry literacy in PMTs is conceptualized as an integrated and interrelated competency rather than a set of isolated skills. For PSTs, the ability to model a geometric structure, define its properties, and recognize its relevance in daily life are viewed as cognitively interconnected processes. Consequently, imposing a multidimensional structure solely to enhance statistical indicators might have introduced an artificial segmentation of these closely related competencies. From this perspective, the single-factor structure – despite explaining a moderate proportion of variance – offers a theoretically coherent and ecologically meaningful representation of professional geometry literacy. Although the RMSEA value (.10) exceeded conventional thresholds for close model fit, structural equation modeling literature emphasizes that overall model evaluation should rely on multiple fit indicators rather than a single criterion. In the present study, incremental fit indices (CFI, IFI, and NFI) consistently demonstrated strong comparative fit, supporting the adequacy of the single-factor model. The decision to retain this structure without introducing post-hoc error covariances was guided by theoretical considerations and the principle of parsimony. Nevertheless, the elevated RMSEA suggests that alternative structural representations (e.g., bifactor or higher-order models) may be examined in future studies to further investigate the dimensionality of geometry literacy.

Specifically for single factor designs, a variance of 30% or higher is regarded as sufficient (Büyüköztürk, 2020; Çokluk et al., 2010). While low variance can be a concern, Schermelleh-Engel et al. (2003) imply that model adequacy is best judged through a holistic assessment of fit indices (e.g., RMSEA, CFI), rather than relying on a single metric. In this study, the robust fit indices obtained from CFA (e.g., CFI = .95; NFI = .94), combined with strong factor loadings and high internal consistency ($\alpha = .94$) (Tabachnick & Fidell, 2001), confirm that the GLS is a valid and reliable instrument despite the moderate variance explained. When compared with similar studies in the literature, some similarities and differences emerge. For instance, Baypınar (2017) developed a mathematical literacy perception scale for seventh-grade students with a four-factor structure and 30 items, indicating that multi-dimensional structures may be preferred in scales targeting younger age groups. Similarly, Katrancı and

Şengül (2019) developed a single-factor scale with 34 items for middle school students, with an explained variance of 47.54%. Although this value is higher than that of the current study, factors such as age level of the target population and the scope of the construct being measured may account for this difference. Bekdemir and Duran (2012) designed a three-factor scale with 38 items to measure primary school students' self-efficacy perceptions regarding VML, reporting a total explained variance of 41.81%. While these studies adopted multi-factor structures, the single-factor structure in the present study demonstrated sufficient explanatory power. Moreover, İlhan (2015) developed a VML perception scale for PMTs comprising five sub-factors and 37 items. Although İlhan's study is similar in terms of target population, the current study specifically focuses on geometry literacy and offers a more concise structure. In this context, the developed scale stands out as an original and psychometrically sound instrument that provides a holistic assessment of PMTs' geometry literacy, contributing to the literature in this field. It is important to note that the GLS is a self-report measure reflecting perceived competencies rather than direct performance. Accordingly, the findings should be interpreted as indicators of perceived geometry literacy rather than objective mastery. While self-perceptions do not equate to actual competence, research in teacher education suggests that efficacy beliefs significantly influence instructional decisions, persistence, and classroom practice. Therefore, although the GLS does not provide a direct assessment of geometry performance, it offers valuable insight into the professional readiness of PMTs from a motivational and self-regulatory perspective. However, previous research confirms the predictive validity of such perceptions; for instance, İlhan and Aslaner (2021) demonstrated that VML perceptions significantly predict actual geometry performance. Thus, the scores obtained from the GLS can be interpreted as valid indicators of PMTs' potential to demonstrate geometry literacy in their future teaching practices.

While the initial conceptual framework presented in Figure 1 outlined distinct characteristics such as modeling, decision-making, and explaining relationships, the empirical results consistently pointed to a unidimensional structure. This finding suggests that for PMTs, geometry literacy acts as a holistic and integrated competency rather than a fragmented set of skills. Unlike primary or middle school students, for whom these cognitive processes might develop separately, PMTs likely utilize these skills simultaneously when engaging with geometric content. Consequently, the single-factor structure aligns with the scientific

principle of parsimony, providing a robust and unified measure that reflects the interconnected nature of geometric thinking at the higher education level.

In addition to exploratory analyses, CFA was conducted for the GLS, and the results confirmed the structure and item composition of the scale. Regarding reliability, Büyüköztürk (2020) states that a Cronbach's α coefficient of .70 or above is considered acceptable. In this study, the reliability coefficients calculated from both the first and second data sets were found to be high, indicating that the GLS has a strong internal consistency and a high level of reliability. When compared with previous studies in the field, similar findings emerge. For example, Baypınar (2017) reported a Cronbach's α of .925 for his scale developed for seventh-grade students. Similarly, the scale developed by Katrancı and Şengül (2019) for middle school students demonstrated a very high reliability coefficient of .97. Bekdemir and Duran (2012) also reported a strong internal consistency with a Cronbach's α of .94 in their study. Although the reliability coefficients in those studies are slightly higher, the values obtained in the present study are within the acceptable and strong reliability range, especially considering the scale's single-factor structure and target group of PMTs. Consequently, the GLS can be regarded as a psychometrically robust instrument that reliably measures geometry literacy in PMTs.

Conclusion

In conclusion, this study addresses a critical gap in mathematics education literature by developing and validating the GLS specifically for PMTs. Rigorous statistical analyses confirmed that the 32 item GLS is a psychometrically sound instrument with high internal consistency and a stable single-factor structure. This one-dimensionality suggests that geometry literacy manifests as a holistic and integrated competency for future teachers, distinguishing it from the multi-dimensional structures often observed in younger student groups. By providing a standardized and reliable measure, the GLS equips teacher educators and researchers with a vital tool to evaluate the geometric readiness of PMTs, identify curriculum needs, and ultimately contribute to the enhancement of geometry instruction in schools.

Limitations

Despite the robust psychometric properties of the GLS, several limitations should be acknowledged. First, the study was conducted exclusively with PMTs in Türkiye. Therefore, the generalizability of the findings to different cultural contexts or other teacher groups (e.g., in-service teachers or primary school teachers) may be limited. In addition, although the scale demonstrated satisfactory psychometric properties within the Turkish context, cross-cultural validation and measurement invariance were not examined. Constructs related to literacy and self-perceived competence may be shaped by cultural, curricular, and educational differences. Therefore, the factorial structure and measurement equivalence of the GLS should be tested across diverse cultural and educational contexts before broader generalizations are made.

Second, the data were collected through a self-report instrument. While self-efficacy and perception scales are valuable, they reflect participants' perceived competencies rather than their actual performance in geometry; thus, response bias cannot be entirely ruled out. Accordingly, the findings should be interpreted as indicators of perceived geometry literacy rather than objectively measured competence. Although self-efficacy beliefs are theoretically linked to instructional behavior and professional practice, they do not necessarily correspond to actual geometry performance. Future studies may integrate performance-based assessments alongside self-report measures to provide a more comprehensive evaluation of geometry literacy. Third, the study employed a cross-sectional design, collecting data at a single point in time. Consequently, it does not capture how geometry literacy evolves throughout the teacher education program. Fourth, although the single-factor structure is stable and valid, the explained variance (36.60%) suggests that geometry literacy is a complex construct potentially influenced by other unmeasured variables. Finally, although the majority of fit indices (CFI, NFI, IFI) indicated a robust model fit, the RMSEA value (.10) remained at the upper threshold of acceptability. Future studies may test the model in different sample groups to verify whether this value is sample-specific or consistent across populations.

Implications and Future Directions

The findings of this study carry significant theoretical and practical implications for mathematics teacher education. Theoretically, the confirmation of the single-factor structure suggests that geometry literacy in PMTs operates as a holistic and integrated competency rather than a fragmented set of skills. This implies that teacher education programs should move beyond teaching geometric definitions, modeling, and problem-solving as isolated units. Instead, instructional designs that integrate these components simultaneously – such as modeling-based activities requiring conceptual explanation – would be more effective in fostering geometry literacy. Practically, the GLS serves as a robust diagnostic tool for teacher educators. It can be administered at the beginning of the program to identify baseline proficiencies or prior to practicum periods (teaching practice) to ensure candidates possess the necessary geometric depth to teach effectively. The scale's high reliability allows curriculum designers to evaluate the effectiveness of current geometry courses and implement remedial interventions where candidates show deficiencies in “geometry literacy”.

Regarding future research, several avenues are recommended. First, since this study is cross-sectional, longitudinal studies could be conducted to track the development of geometry literacy from the freshman year to graduation. Second, while the current study focused on mathematics teachers, extending the scale to pre-service primary (classroom) teachers is crucial, given their foundational role in shaping students' early geometric concepts. Finally, experimental studies could be designed to investigate whether improving PMTs' geometry literacy scores via the GLS directly correlates with their actual teaching performance and instructional quality in real classroom settings. Future research should also examine the cross-cultural validity of the GLS by testing measurement invariance across different countries and educational systems. Establishing configural, metric, and scalar invariance would provide stronger evidence for the generalizability of the construct and enable meaningful comparisons across contexts.

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