

ChatGPT's Knowledge in Mathematics Teaching: An Example of Rational Numbers

Hatice Aydan KAPLAN¹, Melahatnur SARI², Betül DANIK³

¹ Gazi University, Gazi Faculty of Education, Department of Mathematics Education

^{2,3} Gazi University, Graduate School of Educational Sciences, Mathematics and Science Education

ABSTRACT

This research examines ChatGPT's content knowledge (mathematics) and pedagogical content knowledge on the concept of rational numbers. The study was conducted using a case study model within the framework of qualitative research methods. Data were collected by posing open-ended questions to ChatGPT-4 and ChatGPT-4.0 versions, based on the new curriculum objectives set by the Ministry of National Education of Türkiye in 2024. The findings indicate that ChatGPT may be adequate for teaching rational numbers in line with the knowledge required in middle school courses; however, certain limitations in its pedagogical content knowledge were identified. Specifically, it was observed that ChatGPT's pedagogical responses were limited in concepts such as representing rational numbers on the number line and converting them to decimal forms. The research emphasizes that while ChatGPT may not replace teachers, it could play a supportive role in teaching processes, and its integration into education should be carefully considered.

Keywords: Mathematics Education, ChatGPT, Pedagogical Content Knowledge, Rational Numbers, Artificial Intelligence.

INTRODUCTION

Each art has its own rules; however, not everyone who applies these rules can be called an artist, as art emerges when those who know the rules learn how to use them appropriately in the situations they encounter. In this context, teaching is also an important art; decisions about what to do, how to do it, with whom, and at what pace constantly present themselves to teachers throughout a school day, and this varies every day and with each group of students (Schwab, 1983). To perform this art, teachers need to have an in-depth knowledge of both subject matter and its pedagogical applications. This knowledge is defined in the literature as Pedagogical Content Knowledge (PCK). PCK requires that teachers have both knowledge of the subject area and pedagogical skills that make this knowledge understandable for students, a concept rooted in Shulman (1986, 1987). Shulman (1986) defines PCK as the knowledge teachers possess on how to teach topics most effectively, including making explanations in ways that make the subject more understandable, using analogies and examples. According to these definitions, for a mathematics teacher to conduct effective teaching, they must not only possess content knowledge but also know their students, understand their prior learning, anticipate where they may face difficulties, be aware of possible misconceptions, and use teaching methods appropriate to developmental levels to prevent such misconceptions (Shulman, 1986, 1987; Grossman, 1990; Van Driel et al., 1998; Hill et al., 2004; Ball et al., 2008). In the twenty-first century, it is essential for teachers to keep up with technological advancements brought by the times to conduct effective teaching. Among these advancements, artificial intelligence technologies have

gained significant attention in recent years. As recommended by the National Council of Teachers of Mathematics (NCTM, 2014), the adoption of new technologies in the classroom can elevate the standard of mathematics education and promote equity in education. The literature (Hwang & Tu, 2021; Li et al., 2023; Pedro et al., 2023) suggests that artificial intelligence can optimize learning processes by providing personalized support to students, reduce the workload on teachers, assist in the creation of instructional materials, and enrich the learning experience by presenting problems tailored to students' interests. However, it also highlights challenges, such as the potential for artificial intelligence to produce incorrect or misleading responses and reflect biases present in society.

Corresponding Author e-mail: aydane@gazi.edu.tr

https://orcid.org/0000-0002-2036-6996

How to cite this article: KAPLAN HA, SARI M, DANIK B. ChatGPT's Knowledge in Mathematics Teaching: An Example of Rational Numbers, Pegem Journal of Education and Instruction, Vol. 15, No. 2, 2025, 63-75

Data Availability: The data that support the findings of this study are available from the authors, upon reasonable request.

Conflicts of Interest: The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

DOI:10.47750/pegegog.15.02.07

Received: 17.07.2024

Accepted: 20.11.2024

Published: 01.04.2025

Artificial intelligence is a technology with the potential to impact various societal sectors, and as its use becomes more widespread, it is anticipated that demand in certain professions will decline (Timms, 2016). This situation has raised concerns that artificial intelligence might replace teachers (Dumlu et al., 2024). However, studies indicate a strong belief that artificial intelligence will not replace teachers; rather, it will play a supportive role in their work. For instance, Chen and Lin (2023) emphasize that while artificial intelligence can support students' educational development, it cannot replace teachers in providing emotional support and social interaction.

In conclusion, it can be said that there is a consensus that the role of artificial intelligence in education is to support and enrich teachers' work rather than replace them. ChatGPT, one of the most popular AI language models, stands out in this context. Taani and Alabidi (2024) suggest that ChatGPT could serve as a virtual teaching assistant by providing instant access to educational resources and materials. In this case, it is clear that for ChatGPT to effectively assist in teaching, it must possess both content knowledge and PCK comparable to that of a teacher. This study focuses on this aspect of ChatGPT and aims to determine how sufficient this technology is in terms of content (mathematics) and PCK.

ChatGPT and Mathematics Education

ChatGPT, short for "Chat Generative Pre-Trained Transformer," is an artificial intelligence model developed by OpenAI. This model is a large language model with the capacity to generate human-like text (OpenAI, 2022). Using natural language processing and deep learning techniques, ChatGPT can generate logical and contextually appropriate responses to inputs from users (Dempere et al., 2023; Rahman & Watanobe, 2023). Among its potential applications in education are creating lesson plans, developing exam questions, and providing student support services (Taani & Alabidi, 2024). However, there are also several challenges associated with using this tool, such as risks related to transparency, accountability, malicious use, ethical concerns, and plagiarism (Baidoo-Anu & Ansah, 2023; Gouia-Zarrad and Gunn, 2024; Guo et al., 2023; Susnjak, 2022). It is also noted that ChatGPT can produce responses known as hallucinations, which may appear plausible but lack contextual understanding or coherence (Bang et al., 2023).

ChatGPT has been the subject of various studies in the field of education and has been used in different ways. For example, a study by Cooper (2023) examined the potential use of ChatGPT in the context of science education. The researcher investigated how ChatGPT responded to tasks such as preparing exam questions and developing rubrics.

The study concluded that such materials should be carefully evaluated by teachers and adapted to suit the teaching environment. Ellis and Slade (2023) explored the potential of ChatGPT as a tool in teaching data science and statistics. They suggested that ChatGPT could assist in developing course content and provide resources to students in statistical programming classes. Loos et al. (2023) categorized ChatGPT's potential uses in education as answering questions, offering personalized learning opportunities for students, creating educational materials, and language learning.

Various studies on the use of ChatGPT in mathematics education have highlighted the potential benefits and challenges of this AI model across different areas. For example, Frieder et al. (2023) found that ChatGPT's mathematical abilities fall below those of an average graduate student in mathematics. Korkmaz Güler et al. (2024) asked questions from a national mathematics exam to both ChatGPT-3.5 and ChatGPT-4 versions, finding that ChatGPT-4 was more successful in answering the questions and that both versions made common and unique errors. The study suggests that ChatGPT could be used for acquiring basic knowledge and receiving supervised assistance in mathematics education. Wardat et al. (2023) noted that the accuracy of ChatGPT's solutions in geometry-related complex equations may vary. It was emphasized that misunderstandings could occur in certain topics, necessitating oversight by human experts. Guo et al. (2023) examined how closely ChatGPT's responses align with those of human experts, focusing on ChatGPT's distinctive features (such as its tendency to provide long and detailed answers) and the differences between human and ChatGPT responses (e.g., ChatGPT tends to offer objective responses, while humans prefer subjective expressions).

Studies on the use of ChatGPT in mathematics education have also examined how this AI model can contribute to teachers and preservice teachers. For example, in a study by Tapan-BROUTIN (2023), the initial experiences of preservice mathematics teachers with ChatGPT were investigated. It was observed that the preservice teachers used ChatGPT to ask questions about scientific knowledge and teaching strategies, perceiving the interaction with AI as a natural process. In the study by Dumlü et al. (2024), the lesson planning processes of ChatGPT-3.5 and ChatGPT-4 on the topic of inequalities in a mathematics course were analyzed. The study found that both models could assist teachers in lesson planning and easing daily routines. However, it was noted that ChatGPT-3.5 offered more teacher-centered plans, while ChatGPT-4 created student-centered plans. In another study, Urhan et al. (2023) examined the interaction between a university student and ChatGPT, focusing on a discussion of the tangent concept in a calculus course. The findings showed that ChatGPT

provided some rational support in the context of epistemic rationality but generally failed to meet the standards required for communicative rationality. This led the student to find ChatGPT's responses somewhat inadequate or unreliable, highlighting the limitations of ChatGPT in mathematical reasoning tasks.

While ChatGPT can be used as a supportive tool in various areas of mathematics education, research shows that this technology cannot replace a mathematics teacher. ChatGPT can assist teachers with tasks such as lesson planning, material development, and providing student feedback; however, it is incapable of fulfilling aspects like one-on-one interaction and emotional intelligence. According to Baidoo-Anu and Ansah (2023), the ability of artificial intelligence to monitor and guide student development lacks the flexibility and intuitive understanding possessed by a teacher and is limited in meeting the needs of a specific student. Additionally, ChatGPT does not have the sophisticated thinking of a teacher. Based on machine learning algorithms and computational processes, ChatGPT relies on statistical patterns and probabilities derived from training data. It operates without access to real-time information or personal experiences, which can lead it to produce incorrect or meaningless responses, especially when faced with ambiguous or out-of-context prompts (Gouia-Zarrad & Gunn, 2024).

In conclusion, it appears that ChatGPT cannot replace teachers; however, it can effectively support them. ChatGPT can assist teachers in areas such as lesson planning, material development, providing explanations, and student feedback (Taani & Alabadi, 2024). To perform these tasks effectively, however, it must possess a depth of PCK and expertise comparable to that of teachers. PCK enables teachers to understand student needs and select appropriate teaching strategies, raising questions about the extent to which ChatGPT possesses such knowledge. In this context, examining ChatGPT's content (mathematics) and PCK is essential to determine its potential as a supportive tool for teachers. Therefore, this research aims to assess ChatGPT's status in these areas of knowledge.

METHOD

Research Design

This study was conducted within a qualitative paradigm. Qualitative research is a flexible and interpretative approach in which the researcher interacts with participants and aims to deeply understand their experiences (Creswell, 2013; Fraenkel & Wallen, 2006). In this context, a case study model was adopted in the study. A case study is a research method used to examine a specific event, person, or process in depth, with the aim of comprehensively addressing the research

subject and revealing details unique to that context (Merriam, 2009; Yin, 2018). This method provides an appropriate basis for examining the knowledge ChatGPT possesses within the scope of PCK.

Data Collection Tools

In this research, ChatGPT-4 and ChatGPT-4.0 were used as data collection tools. ChatGPT-4 is an artificial intelligence language model developed by OpenAI, noted for its natural language processing capabilities. Compared to previous versions, this model has been trained on a larger dataset and has shown significant improvements in language comprehension, reasoning, and text generation (Brown et al., 2020). Additionally, the ChatGPT-4.0 version has a more advanced and optimized algorithm, allowing it to produce more consistent and context-sensitive responses during user interactions (OpenAI, 2024). Rudolph et al. (2023) stated that, unlike other generative AI models, ChatGPT can produce impressive texts within seconds, making it the most advanced chatbot developed to date. In our research, these programs were chosen as data collection tools because there are no similar alternatives with comparable features and because OpenAI only provides access to ChatGPT-4 and ChatGPT-4.0 versions.

A specially prepared form was used to evaluate the PCK of ChatGPT 4 and 4.0 versions (hereafter referred to as ChatGPT 4 versions). The concept of rational numbers, a key idea in mathematics, was chosen for this purpose. In its 2000 report, *Principles and Standards for School Mathematics*, NCTM identified the Number and Operations Standard as one of the five fundamental content standards and a foundational area. In this context, the importance of teaching rational numbers, introduced in the 7th grade, becomes evident. For students to achieve a complete and accurate learning process, they need to have a solid understanding of the definition of rational numbers and be able to make connections between this concept and various contexts (Çevikbaş & Argün, 2017). Similarly, assessing ChatGPT 4 versions' knowledge of this concept, including its definition and connections to different contexts, is also crucial.

The form prepared to measure the PCK of ChatGPT 4 versions regarding rational numbers in this study includes two learning outcomes related to rational numbers for the 7th-grade level, published by the Ministry of National Education (MEB) in 2024. These learning outcomes are stated as "Being able to interpret natural numbers, integers, and rational numbers in various situations" and "Being able to represent the decimal forms of rational numbers in real-life situations" (MEB, 2024). The form contains a total of 20 open-ended questions designed to assess content and PCK related

to these learning outcomes (see Appendix 1). In developing the questions, relevant literature was reviewed, and the experiences of the researchers were taken into account (1 expert in mathematics education, 2 mathematics teachers).

Data Collection

In the data collection, the questions in the form were posed to ChatGPT to obtain the data. Both the free ChatGPT-4 version and the paid ChatGPT-4.0 version accessible through a subscription were used in this process. Before asking the questions, specific prompts explaining the situation were given to the ChatGPT 4 versions. For example, instructions such as “Now, I will ask you to act as a 7th-grade mathematics teacher. I am interested in your mathematical content knowledge and pedagogical content knowledge in teaching rational numbers. Please answer my questions as if you were a math teacher,” were provided. In this way, ChatGPT was guided to respond from the perspective of a mathematics teacher. All questions were repeatedly asked by three different users at different times, and responses were obtained. After observing a high degree of similarity in responses, analysis was conducted on three different response forms that were considered the most comprehensive.

Data Analysis

The data analysis was carried out using the descriptive analysis method. Descriptive analysis is a commonly used method in organizing and interpreting data, allowing researchers to organize data under themes or categories (Yıldırım & Şimşek, 1999). In this study, three researchers independently analyzed the responses provided by the ChatGPT 4 versions. The responses were evaluated in terms of how sufficient they were as if provided by a mathematics teacher and recorded in an Excel table consisting of “appropriate,” “needs improvement,” and “explanations” columns. These independent analyses were then compared, and coding discrepancies were discussed. Subsequently, the researchers reached a consensus on the codings, and the inter-rater reliability coefficient was calculated as 83%. This rate, being over 80%, reflects an acceptable consistency among the researchers (Miles & Huberman, 1994).

Validity and Reliability of the Study

In qualitative research, the concepts of validity and reliability are critically important for the reliability of the findings and the accuracy of the results. Lincoln and Guba (1985) proposed the concept of “trustworthiness” instead of validity and reliability in qualitative research and defined four main components of this trustworthiness: credibility, transferability, dependability, and confirmability.

In this study, credibility was ensured through various methods. By adopting a triangulation approach, the same questions were repeatedly asked to both ChatGPT-4 and ChatGPT-4.0 versions by three different users at different times. This approach enhanced the consistency of the findings by comparing data obtained from different sources and reinforced the reliability of the research findings. Additionally, responses were analyzed and compared by independent researchers, and coding discrepancies were discussed until consensus was reached. This process contributed to the internal consistency of the study.

In terms of transferability, the focus was on the extent to which the findings of this study can be applied to other contexts and samples. The context and processes of the research were explained in detail through comprehensive descriptions. Detailed description provides an important reference for other researchers who wish to determine the degree to which the findings can be transferred to other contexts. Confirmability pertains to how the researchers arrived at the findings and the degree of objectivity of the results. In this study, consensus among researchers was achieved throughout the analysis process, and agreement was established on the coding. The inter-rater reliability coefficient of 83% also supports this confirmability.

FINDINGS

In this study, which aimed to determine the adequacy of ChatGPT 4 versions in terms of content (mathematics) and pedagogical content knowledge, two different situations emerged. The first situation includes data indicating that these versions have sufficient content knowledge; the second situation includes data suggesting that their PCK is mostly sufficient but somewhat limited. These data are presented along with examples of the questions asked by the researchers to ChatGPT and its responses.

Case 1: The content knowledge of ChatGPT 4 versions regarding rational numbers is sufficient.

The analysis of the data revealed that the content knowledge of ChatGPT 4 versions regarding rational numbers is sufficient. In this context, the responses in Table 1 can be examined.

As seen in the examples in Table 1, ChatGPT 4 versions have defined a rational number as the ratio of two integers and highlighted that the denominator must be a non-zero integer. It has indicated that the numerator and denominator do not need to be co-prime and has appropriately demonstrated the representations of negative rational numbers. Despite the different representations of negative rational numbers, the

Table 1: Responses of ChatGPT 4 Versions Regarding Content Knowledge of Rational Numbers

Sample Questions	Responses of GPT-4 Versions
What is a rational number?	A rational number is a number that can be expressed as the ratio of two integers (numerator and denominator), where the denominator must be non-zero. For example, $\frac{1}{2}$, $\frac{3}{4}$, and 5 (which is equal to $\frac{5}{1}$) are rational numbers.
Rational numbers are expressed in the form $\frac{a}{b}$, as you know. Must a and b be co-prime in this case?	Rational numbers are expressed in the form $\frac{a}{b}$, and a and b do not have to be co-prime. However, if a and b are not co-prime, the fraction can be simplified and expressed in its reduced form.
How are negative rational numbers represented?	<p>Negative rational numbers can be represented by placing the negative sign in the numerator, denominator, or in front of the fraction. The important thing is to indicate that the entire fraction is negative. Here are three different forms of representation:</p> <ol style="list-style-type: none"> Negative Denominator: The negative sign is placed in the denominator. For example, $\frac{1}{-2}$. Negative Numerator: The negative sign is placed in the numerator. For example, $\frac{-1}{2}$. Negative Sign in Front of the Fraction: The negative sign is placed in front of the entire fraction. For example, $-\frac{1}{2}$. <p>Note: The most common mathematical convention is to place the negative sign in front of the fraction ($-\frac{1}{2}$). This is a more common and generally clearer way to emphasize that the fraction is negative. However, the other two representations are also mathematically correct and convey the same meaning.</p>
Does every rational number have a decimal representation? If so, how can we convert rational numbers to decimal form?	<p>Yes, every rational number has a decimal representation. Rational numbers are numbers obtained by dividing a numerator by a denominator, and the result of this division can be expressed in decimal form. Decimal representations can appear in two different forms: terminating decimals and repeating decimals.</p> <p>Converting Rational Numbers to Decimal Form:</p> <ol style="list-style-type: none"> Terminating Decimal Representation: If the denominator of a rational number contains only factors of 2 and 5, which are powers of 10, the number has a terminating decimal representation. For example, to convert the rational number $\frac{3}{4}$ to decimal form, we calculate how many times the numerator can be divided by the denominator: In this process, even though 4 does not divide evenly into 3, by adding enough zeros, we obtain a terminating decimal value. Repeating Decimal Representation: If the factors of the denominator include numbers other than 2 or 5, the resulting decimal representation will be repeating (periodic). For example, if we convert the rational number $\frac{1}{3}$ to decimal form: Here, the digit 3 repeats continuously, which is why this decimal representation is called repeating. <p>Now let's move on to an interesting question: Is the number $0.999\ldots$ equal to 1? To understand this, we'll use some mathematical proofs and logical explanations.</p> <p>Proof 1: Fractional Proof</p> <ol style="list-style-type: none"> Define x: $x = 0.999\ldots$ Multiply with 10: $10x = 9.999\ldots$ Subtract: $10x - x = 9.999\ldots - 0.999\ldots$ $9x = 9$ Find x: $x = \frac{9}{9}$ $x = 1$
Is the number $0.999\ldots$ equal to 1 ?	

Sample Questions	Responses of GPT-4 Versions
	<p>As a result of these operations, we see that .</p> <p>Proof 2: Proof Using Fractions</p> <ol style="list-style-type: none"> Sum of Fractions: <ol style="list-style-type: none"> Let's write as an infinite series: Series Sum: <ol style="list-style-type: none"> This is a geometric series, and its sum can be found using the formula $\frac{a}{1-r}$. First term 0.9, common ratio $r = 0.1$ Sum: $= \frac{0,9}{1-0,1} = \frac{0,9}{0,9} = 1$.

detail regarding the commonly used representation for these numbers is particularly noteworthy. The method it describes for converting rational numbers to decimal form (terminating if there are only factors of 2 and 5 in the denominator; repeating if there are other factors) is a mathematical fact that is not frequently emphasized in the literature. Additionally, it has adequately explained the case of the repeating decimal being equal to 1, with sufficient mathematical justification.

Case 2: The pedagogical content knowledge of ChatGPT 4 versions regarding rational numbers is mostly sufficient, but partially limited.

Questions targeting both content knowledge and PCK were asked to ChatGPT 4 versions. The information under this heading emerged not only from questions related to PCK but also from content knowledge questions. For example, the table below will show responses on how rational numbers can generally be introduced to students (Table 2).

Table 2: Responses of ChatGPT 4 Versions Regarding PCK on Rational Numbers

Sample Questions	Responses of GPT-4 Versions
What is a rational number?	<p>The relationship of rational numbers with natural numbers and integers can be thought of as an extended form of these sets. Natural numbers and integers are special cases of rational numbers. For example, every natural number can also be expressed as a rational number in the form of $\frac{n}{1}$.</p> <p>Understanding rational numbers is crucial for developing mathematical thinking and problem-solving skills, as these numbers are directly related to concepts we frequently encounter in daily life, such as ratios, percentages, and proportions. A solid grasp of these concepts will strengthen students' mathematical reasoning and help them analyze situations they encounter in real life.</p>
As a teacher, how would you define rational numbers to students	<p>When defining rational numbers to students, I would start by connecting to what they already know about numbers. This helps students better understand new information by relating it to prior knowledge. Here are the steps I would follow in defining this concept:</p> <ol style="list-style-type: none"> Reviewing Basic Concepts: First, I would help them refresh their knowledge about natural numbers and integers. I would remind them that natural numbers are zero and positive whole numbers, while integers are an extended set that includes negative numbers. Definition of Rational Numbers: I would define rational numbers as numbers obtained by dividing a numerator by a denominator. The denominator cannot be zero, as dividing a number by zero is undefined in mathematics. Demonstration with Examples: I would explain that these fractions are rational numbers by demonstrating various fractions. For example, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{1}$ (which is equal to 5). I would also show how these fractions are used in different contexts (ratios, percentages, proportions) with examples.

Sample Questions	Responses of GPT-4 Versions
As a teacher, how would you define rational numbers to students?	<p>4. Decimal Representations: By transitioning to the decimal representations of rational numbers, I would explain that each rational number can be expressed either as a terminating decimal or as an infinite repeating pattern (periodic). For example, $\frac{1}{2}$ is 0.5.</p> <p>5. Connections to Everyday Life: I would demonstrate how rational numbers are used in everyday life with examples, such as calculations involving money, measurements in recipes, distance, and speed.</p> <p>6. Visual and Interactive Materials: I would reinforce the concept of rational numbers by using visual and interactive tools such as fraction bars, number lines, and interactive fraction games. These steps help students associate rational numbers with concrete examples and better understand the concept. By receiving feedback from students at each step, I have the opportunity to re-explain any parts they do not understand.</p>
Rational numbers are expressed in the form $\frac{a}{b}$, as you know. Must a and b be co-prime in this case?	<p>In expressing rational numbers in the form $\frac{a}{b}$, it is not necessary for a and b to be co-prime. However, to represent rational numbers in their simplest form, it is generally recommended to simplify by dividing out any common factors between a and b.</p> <p>For example, $\frac{4}{6}$ is a rational number, and in this case, there is a common factor of 2 between 4 and 6. We can simplify this number to $\frac{2}{3}$. This simplification process helps students understand and compare rational numbers more easily, as the form $\frac{2}{3}$ is simpler and more understandable than $\frac{4}{6}$.</p> <p>Additionally, the simplified form of a rational number is often more useful when performing mathematical operations. For example, when adding or multiplying fractions, using their simplified forms can make the calculations easier.</p> <p>In conclusion, a rational number can be formed even if a and b are not co-prime; however, simplification is recommended to make mathematical expressions cleaner and more understandable. This is a common practice in mathematics and an essential skill for students to use rational numbers more effectively.</p>

As can be seen, ChatGPT 4 versions use appropriate explanations when introducing rational numbers to 7th-grade students. Details such as relating rational numbers to integers and natural numbers, as well as their direct relationship with concepts frequently encountered in daily life, like ratios, proportions, and percentages, provide insight into the PCK of ChatGPT versions. Additionally, the headings used, such as “reviewing basic concepts” and “demonstration with examples,” are noteworthy because they can be applied not only in the introduction of rational numbers but also in the teaching of other concepts. The examples provided under the “Visual and Interactive Materials” heading, such as fraction bars, number lines, and interactive fraction games, indicate that ChatGPT 4 versions have knowledge of the elements used to represent rational numbers and recognize the importance of visualization in teaching.

When examining Table 2, it can be seen that the explanations provided by ChatGPT 4 versions regarding the co-primality of the numerator and denominator are sufficient for students. Explaining that the simplest form of a fraction makes calculations easier in mathematics can be considered a satisfactory explanation for 7th-grade students. Similarly, regarding the conversion of repeating decimal representations

into rational numbers, it provided the following explanations that students can understand:

Converting repeating decimal representations into rational number form offers students a great opportunity, especially to develop their mathematical thinking skills and understanding of algebraic manipulations. I use the following methods to teach this process step by step:

1. Introducing Basic Concepts

Explanation: I would explain that repeating decimal representations are sequences of numbers that repeat continuously. For example, $0.\overline{3}$ or $1.\overline{23}$.

Representation: I would emphasize that these types of representations are formed by the repetition of a pattern.

2. Performing the Process Through a Simple Example

Example Selection: I would take a simple repeating decimal representation like $0.\overline{6}$.

Steps:

- Let's assume $x = 0.\overline{6}$.
- If we multiply both sides by 10, we get $10x = 6.\overline{6}$.
- If we subtract the first equation from the second equation: we get, $10x - x = 6.\overline{6} - 0.\overline{6}$.
- We get $9x = 6$.

5. Then, $x = \frac{6}{6} = \frac{2}{3}$.

3. Performing the Process Through a More Complex Example

Example Selection:

Steps:

1. Let's assume $x = 0.\overline{123}$.
2. We multiply the number by to bring the decimal point to the start of the repeating section.
 $10x = 1.\overline{23}$
3. We multiply the number by 1000 so that the repeating section is entirely to the right of the decimal point.:
 $1000x = 123.\overline{23}$
4. We subtract the smaller number from the larger number:
 $1000x - 10x = 123.\overline{23} - 1.\overline{23}$
5. We get, $990x = 122$
6. Then, $\frac{122}{90} = \frac{61}{45}$.

4. Applications and Exercises

I would provide students with various repeating decimal representations and ask them to convert these into rational number form.

We would do group work in class to reinforce the process of converting these types of representations.

5. Using Visual and Interactive Tools

I would use interactive tools or software that demonstrate these conversions on a number line to help students understand the concept visually.

With this methodology, I enable students to both understand and practice the process of converting repeating decimals into rational numbers. This process helps students develop their algebraic thinking and calculation skills.

As can be seen, the explanations provided by ChatGPT 4 versions on this topic are sufficient in terms of mathematical content and are rich in elements such as progressing from simple to complex, exercises, group work, and visual tools. Now, we can look at the data regarding the PCK of ChatGPT 4 versions on the role of zero in rational numbers.

Table 3: Responses of ChatGPT 4 Versions Regarding PCK on the Role of Zero in Rational Numbers

Sample Questions	Responses of GPT-4 Versions
Is zero a rational number? / How would you explain this to your students as a teacher?	<p>Yes, zero is a rational number. When explaining this to students, it's important to clearly and simply convey the reasons why zero is considered a rational number. Here's how I would do it:</p> <p>Dear Students,</p> <p>Today, we will examine whether zero is a rational number. To understand this topic, let's first recall the definition of rational numbers. Rational numbers are numbers expressed as the ratio of two integers and are written in the form $\frac{a}{b}$, where a and b are integers and cannot be zero. Now, let's think about whether zero is a rational number. Can we express zero as a rational number? Yes, we can. Zero can be written as a fraction with a denominator of 1. The integer zero can be written as $\frac{0}{1}$. Here, 0 and 1 are integers, and the denominator is not zero. Similarly, $\frac{0}{2}$, $\frac{0}{3}$, and $\frac{0}{4}$ all represent zero and are considered rational numbers.</p>
Are numbers with a denominator of rational numbers? / How would you explain this to a student as a teacher?	<p>A number with a denominator of zero is not a rational number. This is an important rule in mathematics, and an expression in the form of $\frac{a}{0}$ is undefined. When explaining this concept to 7th-grade students, I would use concrete examples and visual explanations to help them understand why such a restriction exists. Here is a roadmap for how I could convey this information:</p> <ol style="list-style-type: none"> 1. Explaining the Concept of Undefined: I would explain to students that every operation in mathematics has a definition, and some operations cannot be performed or are meaningless, such as division by zero. 2. Why the Denominator Cannot Be Zero: I would explain why a denominator of zero is unacceptable. In a division operation, the denominator represents the quantity being divided by. If the denominator is zero, it means the division is being done by "nothing," which is logically impossible.. 3. Examples from Everyday Life: I would reinforce this concept with examples from everyday life. For instance, imagine a pizza that you are trying to divide among zero people. Since there is no one to divide the pizza for, the division operation becomes meaningless.

Sample Questions	Responses of GPT-4 Versions
Are numbers with a denominator of rational numbers? / How would you explain this to a student as a teacher?	This process shows students that the concept of division by zero is not only a mathematical rule but also logically meaningless. These explanations help students understand and accept this mathematical rule better.

As seen in Table 3, ChatGPT 4 versions explained whether zero is a rational number by referring to the definition of rational numbers. The explanation, "The expression fits the definition of rational numbers because the numerator () and the denominator () are integers, and the denominator is not zero," is sufficiently satisfying for 7th-grade students in understanding why zero is a rational number. In the continuation of the teaching, it attempted to make learning more effective by providing other examples that can represent zero.

When examining ChatGPT 4 versions' explanations on why the denominator of a rational number cannot be zero, as shown in Table 3, it can be observed that a mathematical

fact has been adapted to the students' level. Supporting this concept with details like the meaninglessness of division and the pizza example can be considered a well-intentioned effort. Moreover, pointing out that "showing that the concept of division by zero is not only a mathematical rule but also logically meaningless" would help "students better understand and accept this mathematical rule" indicates the quality of its pedagogical knowledge on this topic.

Alongside all these positive responses, ChatGPT 4 versions also provided some explanations that were more limited compared to its other responses. Table 4 can be provided as an example of these explanations:

Table 4: Limited Responses of ChatGPT 4 Versions Regarding PCK on Rational Numbers

Sample Questions	Responses of GPT-4 Versions
Can you explain how we can represent a rational number on the number line?	<p>To represent a rational number on the number line, we express it as a fraction and then place this fraction in the appropriate position. Now, let's learn how to do this step by step.</p> <p>Steps:</p> <ol style="list-style-type: none"> 1. Identify the Fraction: For example, we want to represent the rational number . 2. Draw the Number Line: Draw a horizontal line and label the midpoint as . Write the positive integers on the right side and the negative integers on the left side. 3. Divide According to the Denominator of the Fraction: The denominator () gives us the units by which to divide the fraction. For example, for , since the denominator is , divide the space between and into equal parts. 4. Mark the Numerator of the Fraction: The numerator () tells us how many parts to move forward. For example, for , start at and move units to the right. 5. Place the Point: Place a dot on the third section and label this point as . <p>Examples:</p> <ol style="list-style-type: none"> 1. Positive Rational Number: Divide between and on the number line into equal parts. Place a point on the third part and label it as . 2. Negative Rational Number: Divide the space between and on the number line into equal parts. Place a point on the second part and label it as . 3. Integer: Place a point at the position of on the number line and label it as .

Sample Questions	Responses of GPT-4 Versions
Does every rational number have a decimal representation? If so, how can we convert rational numbers to decimal form? / How would you explain this to a student as a teacher?	<p>For the first two parts of the question, see Table 1.</p> <p>Explanation to the Student as a Teacher: Converting rational numbers to decimal form essentially means expressing these numbers through division. To show students this process, I can follow these steps:</p> <ol style="list-style-type: none"> Demonstration with Examples: I would present students with examples of rational numbers that have both terminating and repeating decimal representations, and demonstrate the calculation of each step-by-step. Division Practice: We would practice converting various rational numbers to decimal form by performing division exercises together on the board or on paper. Visual Aids: I would use a number line or digital tools to visually show students how decimal representations appear. Discussion: I would have students discuss why some decimal representations are repeating and others are terminating. This helps in understanding the topic better. <p>With these methods, we can help students understand the decimal representations of rational numbers and grasp why these representations differ.</p>

In the section on representing a rational number on the number line, it can be observed that ChatGPT 4 versions provided limited responses in terms of PCK. The lack of emphasis on identifying which two integers the given fraction lies between, as well as the absence of differentiation between the placement of negative rational numbers and positive rational numbers, contribute to these limitations. Similarly, there are also limitations in the explanation of converting rational numbers to decimal form. Notably, the responses focused primarily on division, emphasizing obtaining the decimal representation by dividing the numerator by the denominator. However, this conversion can also be achieved by making the denominator a power of 10. Despite repeated questioning, ChatGPT 4 versions did not include responses that addressed these limitations. Therefore, these responses from ChatGPT 4 versions were evaluated as limited in terms of PCK.

DISCUSSION

In this study, the state of ChatGPT 4 versions' mathematical content knowledge and PCK regarding rational numbers was examined. The findings indicate that ChatGPT's mathematical content knowledge on rational numbers is generally sufficient, but there are certain limitations in its PCK.

ChatGPT 4 versions were able to provide accurate and comprehensive answers on topics such as the definition of rational numbers, the representation of negative rational numbers, and the conversion of decimal representations. However, in this study, ChatGPT's content knowledge was limited to the information that might be encountered in middle school-level classes. Therefore, the advanced mathematical

knowledge expected of a mathematics teacher was outside the scope of this study. For example, deeper mathematical concepts such as the algebraic structure of rational numbers and their definition through equivalence relations were not addressed. Nor was the inclusion of natural and integer sets within the set of rational numbers examined in the context of ordered pairs. Wardat et al. (2023) mentioned that while ChatGPT can discuss mathematics, it is argued that it lacks a deep understanding of the subject. Plevris et al. (2023) observed that chatbots generally provide correct solutions to simple arithmetic, algebraic expressions, and basic logic puzzles, but when it comes to more complex mathematical problems and advanced logic tasks, the responses are not always reliable. This study, however, demonstrated that ChatGPT's knowledge of rational numbers may be sufficient only in line with the knowledge that might arise in middle school classes.

Frieder et al. (2023) state that ChatGPT can be effectively used as a mathematical assistant, functioning as a mathematical search engine and information database interface. While leveraging these strengths of ChatGPT, it is also important to recognize its limitations (Cotton et al., 2023). In this study, certain limitations were also identified in the PCK of ChatGPT 4 versions regarding rational numbers. Specifically, it was observed that ChatGPT's pedagogical responses were not sufficiently in-depth when it came to representing rational numbers on the number line and converting them to decimal form. These findings are consistent with other studies in the literature. For example, in the study by Dumlu et al. (2024), it was observed that ChatGPT was weak in graphical representations. As emphasized in the literature, while

AI-based tools have the potential to provide significant support to teachers, they may remain limited in pedagogical tasks (Chen & Lin, 2023).

It is observed that ChatGPT is considered a potential tool in education for enhancing student engagement, promoting critical thinking, and providing personalized assistance (Firat, 2023). However, to fully leverage this potential, it is important to understand the strengths of ChatGPT. This study has highlighted the potential of ChatGPT in terms of its knowledge of rational numbers. Despite all these positive aspects, it should not be forgotten that the primary role in effective use belongs to the teacher. In a study conducted by Noster et al. (2024) with pre-service teachers, it was found that the rate of incorrect responses could increase when participants encountered an incorrect answer generated by ChatGPT. Additionally, it was observed that even with access to ChatGPT, participants were not able to provide correct answers to all tasks. Although this study reveals ChatGPT's knowledge regarding rational numbers, the appropriate use of this knowledge will still depend on teachers. Therefore, it should be considered that the adequacy of ChatGPT's knowledge of rational numbers is not a key element in mathematics teaching.

Future research could explore how AI-based tools like ChatGPT can be used effectively alongside teachers, how the PCK of these tools can be improved, and what role they could play in teaching complex mathematical concepts. It is especially recommended that more studies be conducted on how these tools can handle deeper mathematical knowledge and pedagogical strategies. For example, studies examining to what extent ChatGPT can grasp content related to the algebraic structures of rational numbers and advanced mathematical concepts, and how this knowledge can be used in teaching processes, could fill a knowledge gap in this area. As mentioned in the literature, the integration of AI tools in education has the potential to reduce teachers' workload and promote equity in education (Chen & Lin, 2023; NCTM, 2014). Optimizing these tools to work alongside teachers, considering their limitations, is critical to enhancing the quality of education and fully realizing the potential of AI in education.

REFERENCES

- Baidoo-Anu, D., & Ansah, L. O. (2023). Education in the era of generative artificial intelligence (AI): Understanding the potential benefits of ChatGPT in promoting teaching and learning. *Journal of AI*, 7(1), 52-62.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>
- Bang, Y., Cahyawijaya, S., Lee, N., Dai, W., Su, D., Wilie, B., Lovenia, H., Ji, Z., Yu, T., Chung, W., Do, Q. V., Xu, Y., & Fung, P. (2023). A multitask, multilingual, multimodal evaluation of ChatGPT on reasoning, hallucination, and interactivity. *arXiv*. <https://doi.org/10.18653/v1/2023.ijcnlp-main.45>
- Brown, T., Mann, B., Ryder, N., Subbiah, M., Kaplan, J. D., Dhariwal, P., Neelakantan, A., Shyam, P., Sastry, G., Askell, A., Agarwal, S., Herbert-Voss, A., Krueger, G., Henighan, T., Child, R., Ramesh, A., Ziegler, D. M., Wu, J., Winter, C.,... Amodei, D. (2020). Language models are few-shot learners. *Advances in Neural Information Processing Systems*, 33, 1877-1901.
- Chen, J. J. & Lin, J. C. (2024). Artificial intelligence as a double-edged sword: Wielding the POWER principles to maximize its positive effects and minimize its negative effects. *Contemporary Issues in Early Childhood*, 25(1), 146-153. <https://doi.org/10.1177/14639491231169813>
- Chen, L., Chen, P., & Lin, Z. (2020). Artificial intelligence in education: A review. *Ieee Access*, 8, 75264-75278. <https://doi.org/10.1109/ACCESS.2020.2988510>
- Cooper, G. (2023). Examining science education in ChatGPT: An exploratory study of generative artificial intelligence. *Journal of Science Education and Technology*, 32(3), 444-452. <https://doi.org/10.1007/s10956-023-10039-y>
- Cotton, D., Cotton, P., & Shipway, J. (2023). Chatting and cheating. Ensuring academic integrity in the era of ChatGPT. *Innovations in Education and Teaching International*, 61(2), 228-239. <https://doi.org/10.1080/14703297.2023.2190148>
- Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches* (3rd ed.). Sage Publications.
- Çevikbaş, M., & Argün, Z. (2017). Geleceğin Matematik Öğretmenlerinin Rasyonel ve İrrasyonel Sayı Kavramları Konusundaki Bilgileri (Knowledge of future mathematics teachers on concepts of rational and irrational numbers). *Uludağ University Faculty of Education Journal*, 30(2), 551-581. <https://doi.org/10.19171/uefad.368968>
- Dempere, J., Modugu, K., Hesham, A., & Ramasamy, L. K. (2023). The impact of ChatGPT on higher education. *Frontiers in Education*, 8, 1206936. <https://doi.org/10.3389/FEDUC.2023.1206936>
- Dumlu, B. Ö., Gezer, E., & Yıldız, B. (2024). Eşitsizlik Konusunda ChatGPT ile Hazırlanan Ders Planlarının İncelenmesi (Examination of lesson plans on inequalities prepared with ChatGPT). *Turkish Journal of Educational Sciences*, 22(1), 337-358. <https://doi.org/10.37217/tebd.1338959>
- Ellis, A. R., & Slade, E. (2023). A new era of learning: considerations for ChatGPT as a tool to enhance statistics and data science education. *Journal of Statistics and Data Science Education*, 31(2), 128-133. <https://doi.org/10.1080/26939169.2023.2223609>
- Firat, M. (2023). How ChatGPT can transform autodidactic experiences and open education? *OSF Preprints*, 1-5. <https://doi.org/10.31219/osf.io/9ge8m>
- Fraenkel, J. R., & Wallen, N. E. (2006). *How to design and evaluate research in education* (6th ed.). McGraw-Hill.
- Frieder, S., Pinchett, L., Griffiths, R., Salvatori, T., Lukasiewicz, T., Petersen, P., Chevalier, A., & Berner, J. (2023). Mathematical

- capabilities of ChatGPT. *arXiv*. <https://doi.org/10.48550/arXiv.2301.13867>
- Gouia-Zarrad, R., & Gunn, C. (2024). Enhancing students' learning experience in mathematics class through ChatGPT. *International Electronic Journal of Mathematics Education*, 19(3), em0781. <https://doi.org/10.29333/iejme/14614>
- Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education*. New York: Teachers College Press.
- Guo, B., Zhang, X., Wang, Z., Jiang, M., Nie, J., Ding, Y., Yue, J., & Wu, Y. (2023). How close is ChatGPT to human experts? Comparison corpus, evaluation, and detection. *arXiv*. <https://doi.org/10.48550/arXiv.2301.07597>
- Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal*, 105(1), 11-30. <https://doi.org/10.1086/428763>
- Hwang, G. J., & Tu, Y. F. (2021). Roles and research trends of artificial intelligence in mathematics education: A bibliometric mapping analysis and systematic review. *Mathematics*, 9(6), 584. <https://doi.org/10.3390/math9060584>
- Korkmaz Guler, N., Dertli, Z. G., Boran, E., & Yildiz, B. (2024). An artificial intelligence application in mathematics education: Evaluating ChatGPT's academic achievement in a mathematics exam. *Pedagogical Research*, 9(2), <https://doi.org/10.29333/pr/14145>
- Li, P.-H., Lee, H.-Y., Cheng, Y.-P., Starčić, A. I., & Huang, Y.-M. (2023). *Solving the self-regulated learning problem: Exploring the performance of ChatGPT in mathematics*. International Conference on Innovative Technologies and Learning.
- Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Sage Publications.
- Loos, E., Gröpler, J., & Goudeau, M. L. S. (2023). Using ChatGPT in education: human reflection on ChatGPT's self-reflection. *Societies*, 13(8), 196. <https://doi.org/10.3390/soc13080196>
- MEB. (2024). *Mathematics Course Curriculum*. Retrieved from <http://mufredat.meb.gov.tr/>.
- Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation: Revised and expanded from qualitative research and case study applications in education*. Jossey-Bass.
- Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook* (2nd ed.). Thousand Oaks: Sage
- NCTM. (2000). *Principles and standards for school mathematics*. National Council of Teachers of Mathematics.
- NCTM. (2014). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics.
- OpenAI. (2022). *Introducing ChatGPT*. Retrieved from <https://openai.com>
- OpenAI. (2024). *Introducing ChatGPT-4.0*. Retrieved from <https://openai.com>
- Pedro, F., Subosa, M., Rivas, A., & Valverde, P. (2019). Artificial intelligence in education: Challenges and opportunities for sustainable development. Retrieved from <https://unesdoc.unesco.org/ark:/48223/pf0000366994>
- Plevris, V., Papazafeiropoulos, G., & Jiménez Rios, A. (2023). Chatbots put to the test in math and logic problems: a comparison and assessment of ChatGPT-3.5, ChatGPT-4, and Google bard. *AI*, 4(4), 949-969.
- Rahman, M. M., & Watanobe, Y. (2023). ChatGPT for education and research: Opportunities, threats, and strategies. *Applied Sciences*, 13(9), 5783. <https://doi.org/10.3390/app13095783>
- Rudolph, J., Tan, S., & Tan, S. (2023). ChatGPT: Bullshit spewer or the end of traditional assessments in higher education? *Journal of Applied Learning & Teaching*, 6(1), 1-20. <https://doi.org/10.37074/jalt.2023.6.1.9>
- Schwab, J. J. (1983). The practical 4: Something for curriculum professors to do. *Curriculum inquiry*, 13(3), 239-265.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1-22.
- Susnjak, T. (2022). ChatGPT: The end of online exam integrity? *arXiv pre-print arXiv:2212.09292*.
- Taani, O., & Alabidi, S. (2024). ChatGPT in education: benefits and challenges of ChatGPT for mathematics and science teaching practices. *International Journal of Mathematical Education in Science and Technology*, 1-30. <https://doi.org/10.1080/0020739X.2024.2357341>
- Tapan-Brouin, M. S. (2023). Examination of questions asked by pre-service mathematics teachers in their initial experiences with ChatGPT. *Journal of Uludag University Faculty of Education*, 36(2), 707-732. <https://doi.org/10.19171/uefad.1299680>
- Timms, M. J. (2016). Letting artificial intelligence in education out of the box: educational cobots and smart classrooms. *International Journal of Artificial Intelligence in Education*, 26, 701-712. <https://doi.org/10.1007/s40593-016-0095-y>
- Urban, S., Gençaslan, O., & Dost, Ş. (2024). An argumentation experience regarding concepts of calculus with ChatGPT. *Interactive Learning Environments*, <https://doi.org/10.1080/10494820.2024.2308093>
- Van Driel, J. H., Verloop, N., & de Vos, W. (1998). Developing science teachers' pedagogical content knowledge. *Journal of Research in Science Teaching*, 35(6), 673-695.
- Wardat, Y., Tashtoush, M. A., AlAli, R., & Jarrah, A. M. (2023). ChatGPT: A revolutionary tool for teaching and learning mathematics. *Eurasia Journal of Mathematics, Science and Technology Education*, 19(7), em2286. <https://doi.org/10.29333/ejmste/13272>
- Yıldırım, A., & Şimşek, H. (1999). *Sosyal bilimlerde nitel araştırma yöntemleri (Qualitative research methods in social sciences)*. Seçkin Publications.
- Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). Sage Publications.

Appendix 1. Sample Questions Asked to ChatGPT and an Example Response in Turkish

1. What is a rational number?
2. How would you, as a teacher, define rational numbers for students?
3. Is every integer a rational number? / How would you explain this to a student as a teacher?
4. Is zero a rational number? / How would you explain this to your students as a teacher?
5. Can rational numbers with denominators that are not powers of 10 be represented in decimal form? How would you explain this to a student as a teacher?
6. Where do we encounter the decimal representation of rational numbers in real life?
7. How would you, as a teacher, explain the usefulness of decimal representations in real life to 7th-grade students?
8. How would you explain to 7th-grade students that every rational number corresponds to a point on the number line? Is the number $0.9999\dots$ equal to 1? How would you explain this to 7th-grade students?