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Algebraic Thinking Process of Students with High Mathematical Ability in Solving Linear Equations Based on Cognitive Systems

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Abstract

Algebraic thinking is the ability to generalize about numbers and calculations, find concepts from patterns and functions and form ideas using symbols. It is important to know the student's algebraic thinking process, by knowing the student's thinking process one can find out the location of student difficulties and the causes of these difficulties. This study aims to analyze students' algebraic thinking processes in constructing new knowledge of high-ability students based on the Cognitive System of Marzano's Taxonomy. The subjects in this study were twenty one mathematics teacher candidates who took linear programming courses. The algebraic thought process of prospective instructors in solving linear equation problems is described using a qualitative descriptive technique in this study. The data collection technique starts with giving algebraic thinking questions and interviews/observations. Data reduction, data display, and deriving conclusions are the data analysis techniques employed. The results of the research show that algebraic thinking processes with types *Generasional Representasi Sequential Concrete* students are able to extract conclusions and organize better. Algebraic thought processes with types *Generational Representation of Concrete Random* students are able to build models and form generalizations but their representation is not good enough that they cannot be communicated properly..

Keywords: Algebraic thinking, thought processes, Cognitive Systems, mazano taxonomy.

Introduction

Mathematics at the college level (PT) is very different from mathematics at other levels. (Sari et al., 2017) mathematics in tertiary institutions includes 4 broad insights, namely: arithmetic, algebra, geometry, and analysis. In developing good mastery of lesson concepts, student commitment is needed to give meaning in the independent learning process, including by increasing the desire to seek conceptual relationships between the knowledge they have and what is learned in lectures, and the lecturer acts as a facilitator of the student learning process (Bayu, 2015). The failure of most math teachers to identify students' missing knowledge is a serious concern for prospective teachers to find out/identify students' lack of knowledge in solving math problems (Zuya, E. H., & Kwalat, S. K. 2015).

One area of mathematics that is taught at the university level is algebra. According to Agoestanto et al., (2019) the basic principles of algebra are very necessary so that students can learn algebraic material in the future. Almost all areas of mathematics require algebra as a method for solving problems. According to (Usiskin, 2012), algebra is necessary because algebra is basically one of the principles used to solve problems. Based on several studies related to algebraic thinking, the research subjects are at elementary to intermediate levels, from the results of algebraic thinking students still experience difficulties. Children who lack fundamental algebraic abilities struggle to solve algebraic problems, simplify algebraic equations and expressions, and comprehend quadratic graphs (Mustaffa et al., 2015). The ability to think algebra is very important for students' lives when they graduate from school, as adults, and in their work lives, so that all students must learn algebra (NCTM., 2000). Students' algebraic thinking ability in Indonesia is still low (Jupri et al., 2014). In Indonesia, like many other countries, mathematics teachers, educators, and researchers are faced with various student difficulties in learning algebra (Jupri, et al., 2014). In the last decade, the achievement of Indonesian

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students in mathematics and especially algebra has been very low, as seen from the results of the Trends in International Mathematics and Science Study (TIMSS). At TIMSS 2007, in terms of algebra, Indonesian students ranked 36th out of 48 participating countries (Mullis et al., 2008). At TIMSS 2011, the same achievement was achieved by Indonesian students in 38th position out of 42 participating countries (Mullis et al., 2012). And the latest result, namely TIMSS 2015, Indonesia is ranked 44th out of 49 countries (Nizam, 2016).

These problems become obstacles for some students when they become students at a university. Some research results show that many students experience difficulties with basic algebraic concepts in high school and university (Faizah, 2019; Ikramuddin, 2017; Novotná & Hoch, 2008; Wasserman, 2014; Wasserman, 2017), one of the difficulties is the inability of students to connect basic algebra with university algebra (Cook, 2012; Suominen, 2015; Suominen, 2018; Usiskin, 2001; Wasserman, 2018, Some of the algebraic structure components at the university level are analogous to the algebraic structure components in high school (Novotna & Hoch, 2008), university algebra is a generalization of school algebra that can represent the variables of various mathematical objects, including numbers, vectors, matrices, functions, transformations, and permutations, and in which expressions and placements are formed through operations that make sense for specific objects: addition and addition to matrices, composition for function, and so on (Findell, 2001). Based on initial studies in the field, it was found that students of the Mathematics Education Study Program at a university in Jakarta, the average algebraic thinking ability was still low, this was seen from the results of students with high algebraic thinking skills of 18% of students, in the medium category as many as 36% of students, and 46% of students % fall into the category of Low algebraic thinking ability. Based on the initial study, it was also found that students still had difficulties in working on algebraic problems. Most students have not been able to give the right answer to the problem given in the question. Algebraic thinking is a skill to focus on the relationship between numbers (Kieran, 2004; Kieran, 2018; Venkat, et al. 2018; Widodo, et al. 2018). It involves generalizing arithmetic and providing reasoning related to it, developing mathematical models (mental and formal) in solving algebraic problems, formulating and visualizing patterns and constructions of algebraic language (Dekker & Dolk, 2011; Hendroanto, et al. 2018). According to Kieran (2004) in working on algebraic questions students carry out generational activities, transformational activities, and global meta-level activities.

Based on the preliminary study, it was found that student results tended to be unable to generalize so that in doing the equations it was not quite right so that the conclusions were irrelevant, students tend not to be able to generalize so that in doing the equations it is not correct so the conclusions are irrelevant, then students are also not able to abstract information so students are not able to write mathematical symbols and are not able to apply mathematical modeling concepts. Students' relatively low algebraic thinking skills are hampered in problem representation and the ability to make strategies to solve mathematical problems. Students experience difficulties in describing and analyzing numerical patterns because students do not understand the problems that must be solved in a pattern and do not know how to determine the next pattern as a solution to solving mathematical problems.

Apart from doing modeling in mathematical problems, Another element is the inability to derive inferences from problem circumstances using objects and representations of use such as graphs, tables, and equations (Pratiwi et al., 2018). Other opinions also state that a lack of algebra can lead to difficulties in further studies such as calculus (Müller et al., 2014) and other mathematical skills such as mathematical proof (Mustafa & Derya, 2016) and problem solving (Ferryansyah et al., 2018). Researchers claim that one of the reasons for this difficulty is that algebra classes often focus on manipulating symbols and procedures over, and often at the expense of, understanding content that is more conceptual (Walkoe, J. 2015).

The difficulties experienced by students can be traced so that improvements can be proposed by analyzing students' thought processes. The thinking process itself can be seen or analyzed when students write down the process of writing tests related to algebra material. This is consistent with the statement of algebraic thinking, which is a method for students to generalize mathematical concepts or ideas from an example to collect these generalizations that are articulated in writing or discussion (argumentation) and articulate them according to their age (Quinlan, 2001). The student's algebraic thinking process is expressed through writing in the process of carrying out the written test, it can also be seen that students' mathematical communication is in writing.

There are several ways to explore the process of thinking, in this study explored based on Marzano's taxonomy. Marzano & Kendall (2006) developed a taxonomic model that combines a wide range of factors that affect how students think. The model that was used to build Marzano's Taxonomy not only explains how people determine whether or not to participate in a new activity at the same time, but it also depicts how information is processed once the decision to engage has been made. According to Marzano's Taxonomic Model, there are three types of mental systems: the self system, the metacognitive system, and the cognitive system. The cognitive system is divided into four levels: retrieval, understanding, analysis, and knowledge usage. While the fourth component of the model is knowledge. Each system affects the level of success of students in doing assignments. (Kuswana, 2012) explains that Marzano's taxonomy is a system that classifies results and theory objectives. Marzano's taxonomy can be used to make questions and help measure students' ability to apply a certain level. The low ability to think algebra is also influenced by the level of self-confidence of students in working on problems. Some students feel less confident when they see the questions, so that it has an impact on when working on the questions. Marzano's taxonomy also provides a framework for identifying higher order thinking skills (Irvine, 2017Three systems and domains of knowledge make up Marzano's new taxonomy, all of which are crucial for thinking and learning. The self-system, the metacognitive system, and the cognitive system are the three systems. The self-system determines whether to start a new task or stick with an existing one when given the option; the metacognitive system establishes goals and maintains the level of achievement of those goals; the cognitive system processes all the information it requires; and domain knowledge provides its content (Krathwohl, 2002).

The cognitive abilities of analyzing (C4), evaluating (C5), and producing (C6) were included to this taxonomy (Anderson & Krathwohl, 2001). Learning outcomes have been modified by Anderson and Krathwohl. This is significant since the original Bloom's Taxonomy put a heavy emphasis on describing cognitive processes. In addition, a transition from low order thinking to high order thinking was made in the taxonomic order used to describe thought processes. Marzano and Kendall (2008) created a new taxonomy that consists of 13 markers of higher order thinking in response to Bloom's Taxonomy's shortcomings. This taxonomy also incorporates numerous elements that affect how kids learn at each step so they can improve their capacity for critical thought. The cognitive element may be used as a reference, in accordance with Marzano's Taxonomy, because it can be utilized as a method to assess students' skills in Indonesia, particularly in mathematics.

The Marzano Taxonomy technique is used by students to improve their arithmetic problem-solving abilities (Fita et al., 2019). The findings of a different study, "Thought Processes of Junior High School Students in Solving Mathematical Problems Based on Marzano's Taxonomy" (Winsaputri et al., 2015), indicated that students with high abilities were able to solve problems based on taxonomy Marzano up to the stage of using knowledge, while students with low abilities are able to solve problems up to the stage of applying knowledge. The study "Analysis of Students' Thinking Processes in Solving Story Problems System of Linear Equations of Two Variables Viewed from Marzano's Taxonomy" by Fauziyah (2015) found that only students with high-level abilities were able to solve problems through the stages of Marzano's taxonomy and carry out conceptual thinking processes. Students with average ability may correctly answer issues using Marzano's taxonomy and engage in semi-conceptual thought processes. Additionally, pupils with poor abilities have a lot of trouble solving issues and do not progress through all of Marzano's phases. (Wulandari, 2014) classified his sample of 6 students into three categories—those with high, medium, and poor abilities—for his study on the "Algebraic Thinking Processes of Students Based on Marzano's Taxonomy". His investigation yielded a variety of findings. According to Marzano's taxonomy, students with high aptitudes typically have good problem-solving skills. In contrast, one of the students with average talents was able to solve issues based on Marzano's taxonomy but came up with incorrect answers as a consequence of using the erroneous metacognitive processes. Finally, low-ability pupils come up with incorrect answers since they don't feel pushed by the issues they are faced with.

According to Kuswana (2012), Marzano's taxonomy is a framework for categorizing the outcomes of theoretical models of mental processes and aims. Marzano's taxonomy may be used to create questions and assist teachers in assessing students' capacity to apply a certain level in accordance with their circumstances. According to Fortuna (2018), the Marzano taxonomy's category of cognitive characteristics is more thorough since it interacts with the "three prior knowledges," which are information, mental processes, and psychomotor processes. Marzano The taxonomy describes an issue and is descriptive in nature. The Marzano taxonomy's cognitive components include knowledge processes procedures, remember or do without understanding (retrieval), process sequence or structure of knowledge, synthetic or its steps and descriptionsis the basis of basic understanding or understanding, process access and check knowledge about similarities and differences, diagnosis consequent and predictable errors or logic principle (analysis), process of use knowledge of where problems can occur, and so on. Based on the results of previous research, there is no theory that discusses the spatial thinking processes of students studied based on Marzano's taxonomy in solving algebraic problems. This study conducted an in-depth study to look in detail at students' algebraic thinking processes in solving linear equations based on Marzano's taxonomy, especially the Cognitive System. The purpose of this research is to analyze students' algebraic thinking processes in constructing new knowledge. These results are used for suggestions for improvement in learning linear equation material based on cognitive systems.

Method

Research Design

The qualitative thought processes of prospective instructors in solving Diophantine linear equation problems are described in this study. The natural context, the researcher as the primary instrument, various data sources, inductive data analysis, emergent design, and holistic accounts are all features of qualitative research (Creswell & Plano Clark, 2018). Descriptive research contains data collected and explained in sentences. Based on Marzano's taxonomy cognitive framework, the researcher studied the written and spoken responses to gain data about the sorts of algebraic thought processes used by pupils. Copying data, reducing data by forming abstractions, assembling each item of data and organizing it by coding, creating forms of algebraic thinking processes based on Marzano's cognitive taxonomy system, and drawing conclusions are all phases in data analysis.

Sample and Data Collection

This study was carried out at STKIP Kusuma Negara in the Mathematics Education study program, with second semester students serving as research subjects. This study had twenty-one volunteers as research subjects, three males and eighteen women. Purposive sampling is a sampling approach for data sources that takes specific factors into account (Sukestiyarno, 2020). The study subjects were then chosen using a purposive sampling strategy by two research samples. The selection of the sample in this study is to choose a sample with the final outcome criteria based on algebraic thinking skills. Based on the data collected, from the 21 research subjects selected 6 students with high algebraic thinking skills, 8 students with moderate algebraic thinking skills and 7 low algebraic thinking skills, based on indicators of high algebraic thinking skills obtained 6 students, then based on variations in answers, uniqueness answers and communication skills, from 6 research subjects 2 students were selected to take part in in-depth interviews. The researcher chose 2 out of 6 students to take part in the interview to discuss the algebraic thinking processes that occurred, the reasons for selecting the two subjects were based on the fulfillment of the indicators of algebraic thinking ability. Then the two subjects were coded S1 and S2. The linear equation problem exam is used in this study to collect data and activity records via interviews. Data reduction, data presentation, data interpretation, and generating conclusions were all used in qualitative analysis.

Data Collection

The test is used to collect data on the algebraic thinking ability of prospective mathematics teachers. A description test with measuring material is the sort of test. There are eight exam items in all. In this study, content validity was used to validate the algebraic thinking skills test instrument. The purpose of content validity testing is to determine an instrument's capacity to assess the content (concept) to be measured. Expert judgment is used by researchers to evaluate the level of validity of the instrument to be employed. Experts evaluate the items based on a variety of factors, including the appropriateness of the information, constructions, and language utilized. When working on exam questions, document data in the form of textual test results and documentation images. Interview questions were utilized to clarify the test answer sheets obtained and enhance the coding process (Creswell & Plano Clark, 2018). This data was gathered via smartphone audio, whilst field notes were taken throughout the study procedure.

Data Analysis

The data obtained from the results of the research was carried out by following the procedures of a qualitative model which consisted of collecting data, selecting data, separating data, making analogies, and making hypotheses (Sukestiyarno, 2020). The research subjects were given an algebraic thinking ability test. The researcher separated the data based on test results, which were divided into high algebraic thinking skills, moderate algebraic thinking skills, and high algebraic thinking skills. Then from the test the students' algebraic thinking processes were carried out. The research was conducted using in-depth interviews to deepen the analysis of aspects of algebraic thinking in linear equations courses. The process of drawing conclusions is done by testing the hypothesis by repeating data collection. Triangulation was carried out to validate the data, namely a combination of algebraic thinking tests, in-depth interviews, observations, and documentation. Data reduction was carried out to remove data that was not needed in the study.

Findings

Based on the results of the algebraic thinking skills given to all students, the grouping of algebraic thinking skills is obtained as shown in Table 1.

Table 1. Students' algebraic thinking ability

Number of Students who take the Test	Algebraic thinking skills		
	Height	*Currently*	*Low*
21	6	8	7

Table 1 shows that of the 21 students who were given an algebraic thinking ability test, the results obtained were 6 students having high algebraic thinking skills, 8 students having moderate algebraic thinking skills and 7 students having low algebraic thinking skills. The research subjects were students who had high algebraic thinking skills. High-ability students if the test results obtained are greater than 80 and the maximum score is 100 (Baiduri, 2013). Furthermore, it was found that 2 research subjects were determined from students who had high algebraic thinking skills who fulfilled the components of algebraic thinking and the cognitive system of Marzano's taxonomy. Then the two subjects were coded S1 and S2. S1 algebraic thinking processes with types Generasional Representasi Sequential Concrete and S2 algebraic Thinking Processes with types Generational Representation of Concrete Random.

S1 algebraic thinking processes with types Generasional Representasi Sequential Concrete

Cognitive System Level Retrival Knowledge of S1 subjects which is revealed through conducting interviews, namely by reading and understanding the questions. S1 subjects carry out an analysis by identifying problems in verbal or spoken form. the subject was able to write down all the known information on the problem including the number of basketballs, the number of footballs and each price between basketballs and footballs. Furthermore, the algebraic thinking process of the cognitive system is identified.

At the cognitive system level Retrieval Knowledge, then S1 carries out the process of working on questions in the form of written assignments.

In this cognitive process S1 performs recalling/calling by reproducing the necessary information by knowing what is known and what is being asked in the questions in their own language by identifying questions and applying the questions to the language of mathematics.

S1 also carried out the coordinating procedure through interview tracing. The following are the outcomes of interview tracing coordination.

Q1: What do you think when you are given this question?

S1: Asearch for information then understand the problem by determining what is known then what is being asked sir. Then determine the solution.

Based on Figure 1 on the cognitive system, S1 is able to process information originating from questions well, at level 1 is retrieval knowledge S1 subjects start by understanding and adjusting the rules that must be followed by writing known and asked correctly, S1 subjects identify questions carefully then determine whether his statement is true. In the knowledge retrieval process, the coordination process is carried out in written assignments by making S1 subjects determine what is known and then using their own language to exemplify each variable. So that each variable can be easily understood and searched for solutions. S1 Subject Classifying Information Thinking with the skills of Organizing Information in ways that are useful for uncovering the patterns, relationships, and rules that determine it. This shows that S1 performs the algebraic thinking process well.

At Level 2 Comprehension knowledge, S1 subjects write examples and equations in the correct order or steps, by describing critical aspects of knowledge in the form of symbols and followed by identifying important information. Undergraduate Subjects Visualize problems and translate problems into mathematical language by writing examples and equations in mathematical form, S1 subjects are able to transform question sentences into mathematical models. Subjects identify and place information into various appropriate categories.

The following is the process of working on the S1 followed by coordination which can be traced through interviews.

Q1: What do you do once you have all of the facts in the problem?

S1: Make an example for basketball x and football y to make it easier to solve the problem. In addition, determine each price for each equation. Then determine the solution method.

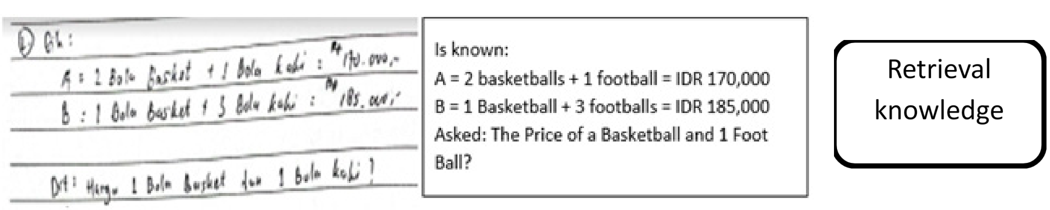


Fig. 1: The results of S1 work in the form of written assignments by exemplifying the variables

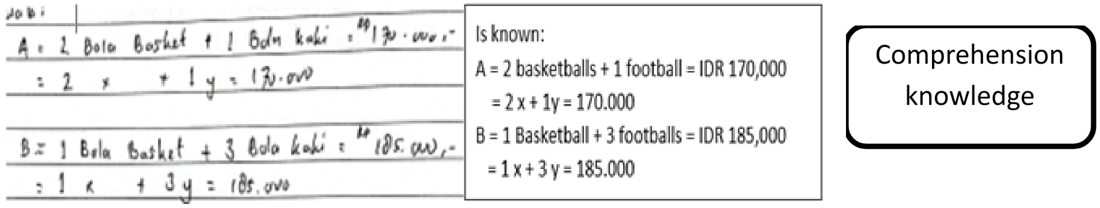


Fig. 2: Results of S1 answers in the form of a mathematical model

Based on Figure 2 and the results of the interview, Subject S1 carries out a mathematical modeling process by eg a basketball with the symbol x, a football with the symbol y and also determines the price. With this step, S1 subjects applied their understanding of linear equations that they had learned so that they obtained 2x+1y = Rp.170,000.00 and 1x+3y = Rp. 185,000.00.

At level 3 analysis of knowledge, S1 Subjects carry out the process of testing knowledge about similarities and differences, relationships, diagnosing strategies that can be used. S1 subjects predict patterns where S1 subjects try to find and understand regularities in certain situations, find patterns from the given problems and then devise general rules based on patterns. S1 performs recalling by describing or dismantling previously possessed knowledge regarding linear equations, namely the use of elimination and substitution methods. From the results of S1's answers, the results of calculations using the elimination and substitution methods are obtained to find the x and y values. so that each value x = 65,000 and y = 40,000 is obtained.

The following is an analysis of knowledge that is traced through interviews.

Q1: What is the purpose of this problem and what method do you use?

S1:To find out the price of a ball pack

Q1: what is the reason for using that method? Try to tell me!

S1: it's faster and easier to do, sir. By way of elimination and substitution makes doing it faster. Even though it is actually biased to use only one.

From the results of the interviews, S1 is able to understand the problems faced, and determine the appropriate solution method to solve the problem. S1 describes the problem solving process on the answer sheet and shows the method chosen clearly and in detail. Explanatory sentences written on the answer sheet show confidence in the chosen strategy. S1 has understood the problem well, students are able to explain the intent and purpose of the problem. This shows that S1 carries out an algebraic thinking process using the correct reasoning process.

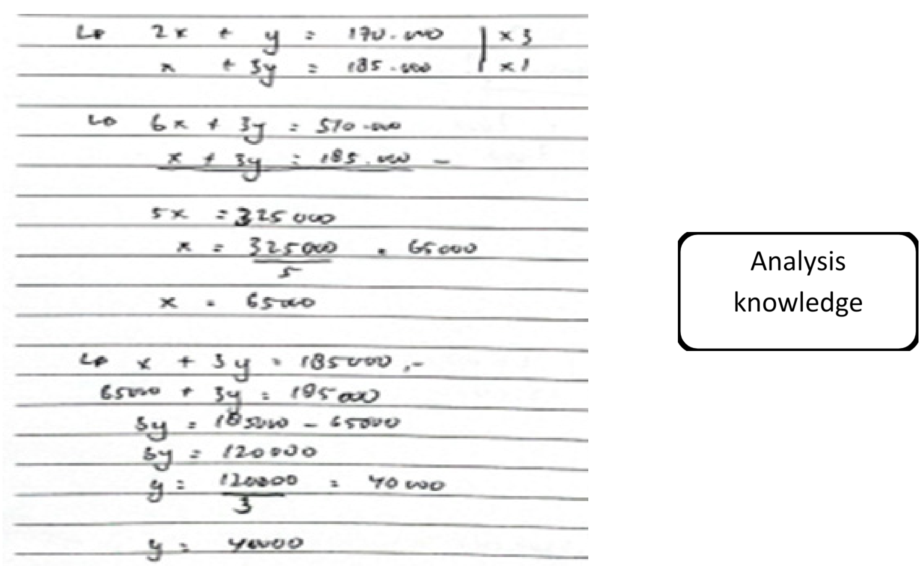


Fig. 3: The results of S1's answers in the form of a sincere assignment using determining the value of x and y

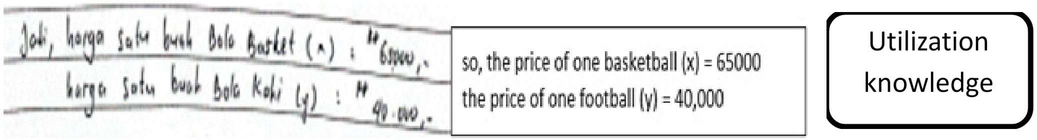


Fig. 4: Conclusion of the final answer

At level 4 utilization knowledge, S1 Able to answer questions and write conclusions correctly. In this process S1 investigates a hypothesis after conducting experiments and collecting data which then concludes new generalizations from known knowledge. S1 expresses in writing by generalizing the results obtained from the price of a basketball and the price of a football. S1 uses inductive and deductive reasoning to explore and find out what is important, making it easier to identify information in obtaining conclusions from the results of solving problems (Utilization of use).

Q1: What can you conclude from the linear equation problem you are working on?

S1: According to Sasya, in determining the x and y variables, a combined method of elimination and substitution was used.

From these basic algebraic concepts, S1 can conclude that the final price of a basketball is Rp. 65,000 and a football is Rp. 40,000 which is the asking price with the right result. These results indicate that S1 carries out algebraic thinking processes by using correct reasoning with arithmetic generalizations. This shows that S1 performs thematization in oral form by making generalizations and is able to conclude the answers to questions correctly. The following is an analysis of algebraic thinking processes by understanding algebraic concepts, reasoning processes and using Sequential Concrete Representation correctly in solving problems based on the cognitive system shown in Table 1.

S2 algebraic Thinking Processes with types Generational Representation of Concrete Random

Based on the results of the S2 subject's work fulfilling the generational ability indicators, the subject wrote down all the known information on the problem. The subject writes down the variables in each question, is able to understand equations that contain variables, namely the equal sign, finds patterns from the problems given and compiles general rules based on patterns. Furthermore, the algebraic thinking process of the cognitive system is identified.

At the cognitive system level Retrieval Knowledge, then Masters will carry out the process of working on questions in the form of written assignments.

In this cognitive process S2 performs recalling/calling by reproducing the necessary information by knowing what is known and what is being asked in the questions in their own language by identifying questions and applying the questions into mathematical symbols.

S2 also carried out the coordinating procedure using interview tracing. The following are the outcomes of interview tracing coordination.

Q1: What did you do when you were first given the question?

S2: Understand the information contained in the problem and the steps taken in solving it.

Table 1: Description of the Algebraic Thinking Process for Each S1 Subject

	System	*Description*
		S1
Cognitive System	Retrieval (knowledge)	Extract information from the problem by determining what information is known and what is being asked
	Comprehension (understanding)	Visualize problems and translate problems into mathematical language by writing examples and equations in mathematical form
	Analysis (analysis)	Test the results of problem identification and solve problems according to the method used.
	Utilization (use)	Write a conclusion from the results of solving the problem

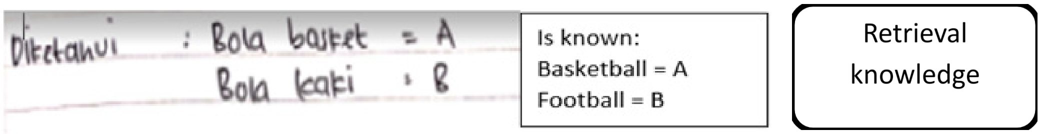


Fig. 5: The results of S2's work in the form of written assignments by exemplifying the variables

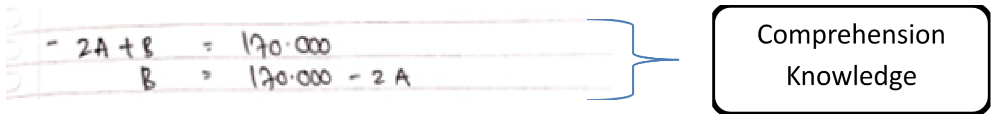


Fig. 6: Results of S2 answers in the form of assignments on comprehension knowledge

Based on Figure 5 on the cognitive system, S2 is able to process information from questions by making each example for basketball with example A and football with example B, but here subject S2 does not write the complete information he gets from the questions. S2 only writes the information needed to make it easier to work on. This shows that S2 performs an algebraic thinking process using Concrete Random Representation.

At Level 2 Comprehension knowledge, in this step Masters directly applies the formula that has been obtained and immediately applies it to find solutions. S2 subjects recalled the formula used by using substitution. These steps are then re-coordinated with subsequent interrelated steps.

Q1:What steps do you take after knowing the information in the problem?

S1:Make an equation with the symbols for basketball A and football B. Then immediately determine the solution.

Based on Figure 6 and the interview findings, S2 carried out the comprehension knowledge process in a written assignment by directly looking for the initial equation using the substitution formula. This shows that S2 recalls the steps in doing what has been learned.

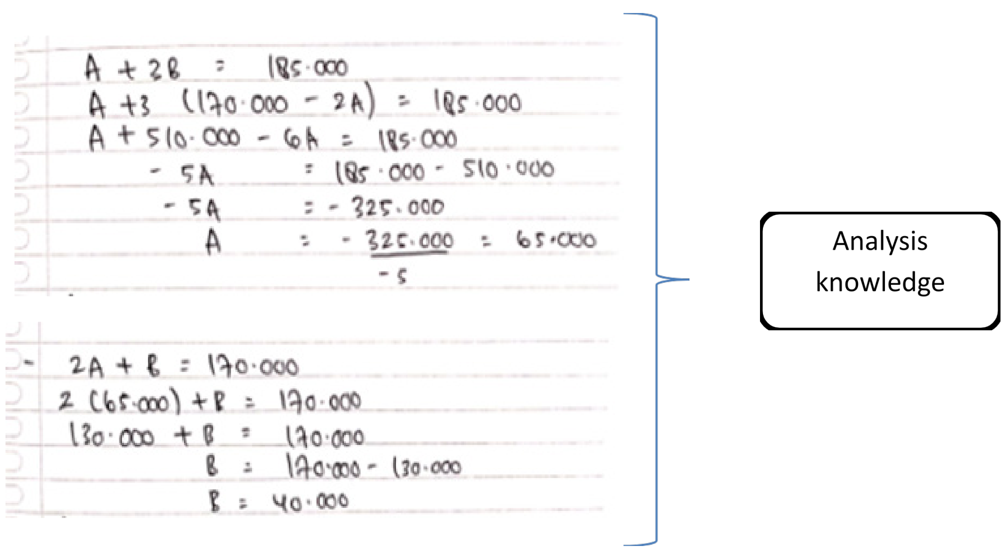
At level 3 analysis of knowledge, Subject S2 carries out the process of testing knowledge about similarities and differences, relationships, diagnosing strategies that can be used. Subject S2 predicts patterns where Subject S2 tries to find and understand regularities in certain situations, finds patterns from the given problems and then compiles general rules based on the patterns. S2 performs recalling by describing or disassembling previously possessed knowledge regarding linear equations, namely the use of substitution. From the results of S2's answers, the results of calculations using the elimination and substitution methods are obtained to find the x and y values. so that each value A = 65,000 and B = 40,000 is obtained

Q1:What is the purpose of this question

S2:As asked sir, looking for the price of a ball

Q1;What method did you use and why did you use that method?

S2;Sir substitution method



Fig/ 7: S2's answers in the form of assignments on knowledge analysis

Q1;Why not just use the elimination or mixed methods?

S2;I worked using a mixed method too, but in my opinion, the method is just faster, sir, in my opinion.

From the results of answers and interviews, Masters was able to understand the problems faced, and determine the appropriate solution method to solve the problem. S2 describes the problem solving process on the answer sheet and shows the method chosen clearly and in detail. Explanatory sentences written on the answer sheet show confidence in the chosen strategy. S2 has an experimental nature by finding alternative answers and doing everything on their own. S2 has understood the purpose of the problem, and chose the substitution method in solving problems on the problem, using the substitution method makes problem solving faster, in this case S2 has understood the problem well, students are able to explain the intent and purpose of the problem. After obtaining the results of variable A, S2 proceeds to the next equation to determine the results of B. This shows that S2 carries out an algebraic thinking process using the correct reasoning process.

At level 4 utilization knowledge, S2 Able to answer questions and write conclusions correctly. In this process S2 investigates a hypothesis after conducting experiments and collecting data which then concludes new generalizations from known knowledge. S2 expresses in writing by generalizing the results obtained from the price of a basketball and the price of a football. S2 uses inductive and deductive reasoning to explore and find out what is important, making it easier to identify information in obtaining conclusions from the results of solving problems (Utilization of use).

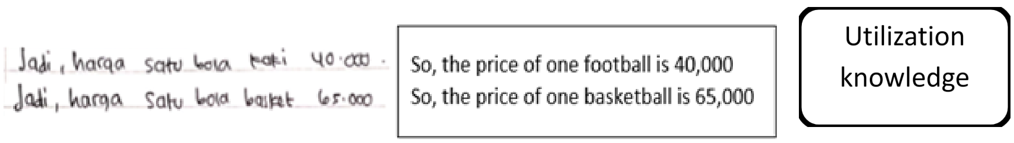


Fig. 7: S2's answers in the form of assignments on knowledge analysis

Q1:What can you conclude from the linear equation problem you are working on? Can you tell me how you do it?

S2:With the initial step of finding the equation to determine the substituted equation I, then proceed to equation 2 to find the respective values of variables A and B.

From these basic algebraic concepts S2 can conclude that the final price of a basketball is Rp. 65,000 and a football is Rp. 40,000 which is the asking price with the right result. These results indicate that S2 performs algebraic thinking processes by using correct reasoning with arithmetic generalizations.

From the results of the interview above, it shows that the S2 self-system is very good in knowing the material contained in the questions and feeling challenged in working on the questions and trying as much as possible to answer the questions correctly.

Based on the results of interviews and tests conducted by researchers on the Masters, it can be concluded that, on the Masters self-system, they know the material in the problem and try to be able to work on the questions well, and on the metacognitive system, Masters has understood the problem and the purpose of the problem and chose the method according to which the solution was the soonest. Furthermore, the S2 Cognitive system is able to complete 3 cognitive levels namely, retrieval knowledge, comprehension knowledge and analysis knowledge. This shows that S1 performs thematization in oral form by making generalizations and is able to conclude the answers to questions correctly.

The following is an analysis of algebraic thinking processes by understanding algebraic concepts, reasoning processes and using Concrete Random Representations correctly in solving problems based on the cognitive system shown in Table 2.

Table 2: Description of the Algebraic Thinking Process for Each S2 Subject

Cognitive System

System	*Description*
	S2
Retrieval (knowledge)	Extract information from the problem by determining what information is known and what is being asked.
Comprehension (understanding)	In translating the problems in the problem into mathematical language not coherently, writing examples correctly but writing equations not with complete steps and structures
Analysis (analysis)	Test the results of problem identification and solve problems according to the method used.
Utilization (use)	Write a conclusion from the results of solving the problem

Discussion

Based on the results above, each S1 and S2 subject fulfills the generational ability indicator, namely representing the problem using an equation that contains something unknown, the subject. S1 is able to determine the variable meaning of a problem, by writing down all the known information in the problem using certain variables. This shows that the subject has the characteristics of denotation (sign), where the subject knows that uncertain numbers must be named or symbolized (Radford, 2014). This stage becomes very important to proceed to the next stage, the more complete that is known in the problem, the easier it is to solve the problem.

S1 is able to complete all cognitive levels namely knowledge retrieval, Comprehension knowledge, knowledge analysis, utilization knowlidge, at this stage the subject fulfills the components of algebraic thinking skills namely Displaying relationships visually (images), symbols, numerically and verbally. S2 subjects are also able to complete 4 cognitive levels, namely knowledge retrieval, comprehension knowledge, analysis knowledge and utilization knowledge. The results of this study indicate that students *Generasional Representasi Sequential Concrete* system and *Generational Representation of Concrete Random*same completes the answer well.

In the cognitive system level retrival knowledge obtained by type student results *Generasional Representasi Sequential Concrete* system and *Generational Representation of Concrete Random*written by writing the components of the problem in the question. At the cognitive level, each subject identified elements that were known and asked to determine each variable. At this stage the two subjects fulfilled the algebraic thinking ability component, namely problem solving ability. The two subjects also fulfill elements of algebraic thinking, namely generalization, which is a process of finding patterns and shapes that begins with the identification of the problems given. On the cognitive system, type *Generasional Representasi Sequential Concrete* coordinate by starting with making an example of a basketball and a foot ball. Then determine the form of the equation, using the example for a football and basketball using the symbols x and y, displaying the relationship symbolically, numerically and verbally. (Dindyal, 2007) revealed that students generally express generalizations verbally before turning to symbolization. (Cañadas and Castro, 2007) distinguish two ways of expressing relationship order. The first way involves the use of symbols and numbers, and the second refers to the use of natural (verbal) language to express generalizations.

Ability *Generasional Representasi Sequential Concrete* interact with deductive reasoning so that S1 subjects with type *Generasional Representasi Sequential Concrete* able to identify the truth in interpreting information in reaching goals or conclusions. While type*Generational Representation of Concrete Random* do examples using symbols then apply operations, analyze problems to be explored, then form hypotheses and build models and generational formation. S2 subjects directly determine conclusions in doing work using the direct substitution method, and provide explanations of answers that are easier to understand in doing work. There is a gap between students' ability to express generalities verbally and their ability to use algebraic notation comfortably (Zazkis, R., & Liljedahl, P., 2002). The introduction of variable notation is important in the development of students' algebraic thinking skills as a basis for building generational (Brizuela, B. M., et al. 2015).

In the cognitive system level Comprehension knowledge of students on type *Generasional Representasi Sequential Concrete* able to carry out the activity of presenting problems in the relationship between variables where the subject is able to make a mathematical model of $2x + y = 170,000$ and $x + 3y = 185,000$, this is in accordance with what is known in the problem, students model in detail on each variable so that nothing is missed. S1 is able to visualize problems and translate problems into mathematical language by writing examples and equations in mathematical form. While students with type *Generational Representation of Concrete Random* present by thinking about ideas and linking previous knowledge to solve problems directly. In this type the subject is different from the steps applied by S1, S2 works in 1 step by directly entering the equation and immediately operating it, so that the work is not done in the correct order, even though the results obtained are the same as type S1 thing. Representation is utilized to aid in the recall, comprehension, reasoning, and communication of objects and connections between items represented in space (Zwartjes, 2018).

In the cognitive system level of student knowledge analysis on type *Generasional Representasi Sequential Concrete* test the identification of the problem by solving the problem according to the application of two methods, namely substitution and elimination. Subjects can implement the formula in solving problem problems. Based on this, the subject can describe the solution using more than one concept to solve the problem, the subject is able to work on the problem in more than one way to solve this, it can be seen from the work that is correct. While students of type *Generational Representation of Concrete Random*can solve the problem on the question. The subject is able to complete the results of the settlement and use one way to solve the problem. Subjects are able to create in using mathematical symbols so that they are able to work on problems or how to solve these things can be seen from their correct work. The reason for this situation is considered to be the fact that students learning arithmetic are oriented towards basic results and that they focus on calculations (Kızıltoprak, A., & Köse, N. Y., 2017), students tend to do calculations (Kieran, 2004). Students generalize properties that only apply to certain conditions in different situations (Biber, Ç., Tuna, A., & Korkmaz, S. 2013).

In the cognitive system level Utilization knowledge of students on type *Generasional Representasi Sequential Concrete* able to organize better in extracting conclusions. While students of type *Generational Representation of Concrete Random* able to use the results of analysis to solve problems but not complete according to the questions in the problem. Cognitive conflicts will arise as students rebuild mathematical systems to make sense of new material. This might result in path-dependent reasoning, in which students may offer multiple solutions to the same topic depending on how they approach it (Pratiwi, V., Herman, T., & Lidinillah, D. A. M. 2018).

Subject S1 type *Generasional Representasi Sequential Concrete* have the motivation to understand the problem, be able to define the problem and understand it well seen from the answers and interviews where they know the problem and are interested in solving the problem, S1 subjects are able to define the problem and understand it well and know clearly the purpose of the question students are faced with problem solving, students can interpret information consciously knowing what strategies can help them in finding solutions, how to implement these strategies, and the reasons for using or choosing these strategies. S1 thinking by holding on to and processing information in an orderly way. They pay attention to and remember reality, easily recalling specific facts, information, formulas, and rules. Notes or papers are a good way for concrete sequential people to learn. The concrete sequential student must organize tasks into a step-by-step process and strive for perfection at each stage. pay attention and remember details more easily, organize tasks in a step-by-step process, and strive for perfection. The stages of the student generalization process are similar to those revealed by (Booker, 2010), that there are 4 stages of inductive reasoning, namely the pattern identification stage, the proof testing stage of the pattern, and the final stage for finding rules for general cases.

Subject S2 type *Generational Representation of Concrete Random*able to define the problem and understand it well seen from the interview answers where they know the problem and are interested in solving the problem, seen from the results of the S2 interview have confidence in solving the problem. A good self-system that will help these students solve math problems (Zaozah et al, 2017). The results of research conducted by researchers are in accordance with the self-system according to Marzano's taxonomy (Muliawati & Istianah, 2017) which states that when the self-system has a positive attitude in a new task, the confidence and motivation to complete the task are also high, so that students are able to complete the new task. well and vice versa. This study supports the findings of (Siregar & Firmansyah, 2021) who found that students with strong cognitive capacities can identify difficulties, plan problem solutions, carry out plans, and review well. Pupils that have strong positive thoughts will be better able to master mathematics and enhance their analytical abilities (Bakar, 2019). In thinking S2 has an experimental attitude accompanied by less structured behavior.

Concrete random thinkers have a strong drive to find alternatives and do things their way. Time is not a priority for Concrete Random people, and they tend to ignore it, especially when they are involved in an interesting situation. They are more process oriented than results, as a result, tasks are often not completed as planned because of the possibilities that arise and invite exploration during the process. So the concrete random thinker holds to reality and has an attitude of wanting to try. Subject S1 type *Generasional Representasi Sequential Concrete* and S2 *Generational Representation of Concrete Random*able to understand the problem well and clearly know the purpose of the problem and choose the right solving method where S1 and S2 choose the method that they think is faster in solving and solving the problem correctly. In solving using different methods or concepts but the results obtained from the two types are the same. Students with a high level of conceptual understanding are able to solve problems in a variety of different forms and settings (Ibrahim, N. N., Ayub, A. F. M., & Yunus, A. S. M. 2020). Represent the idea of generalization algebraically. This is consistent with the results of research on pattern generalization which found that each individual can see the same pattern in a different way (Rivera & Becker, 2007).

Both types fulfill the indicators of algebraic thinking ability, students with type *Generasional Representasi Sequential Concrete* with good language or verbal mastery, are able to present questions in the form of mathematical models in detail so that they can solve problems precisely and completely, while students with*Generational Representation of Concrete Random*students think about ideas by associating previous knowledge in the form of symbols and numerics so that they find the final result correctly even though it is incomplete. This means that the two types of students differ in their algebraic thinking processes but produce the right answers, even though the cognitive level of the two types has differences in cognitive level and type. *Generasional Representasi Sequential Concrete* slightly more complete in comparison *Generational Representation of Concrete Random*.

Conclusion

Based on the results and discussion above, it can be concluded that first, the algebraic thinking process with types *Generasional Representasi Sequential Concrete* S1 with type *Generasional Representasi Sequential Concrete* able to solve algebraic problems as a whole, namely by fulfilling indicators of algebraic thinking, students use their representation abilities verbally, visually, symbolically and numerically and are able to organize better in extracting conclusions. *Representasi Sequential Concrete* characterized by expressing abstract problems in the form of mathematical models properly. so that it can be communicated and resolved completely and correctly. *Representasi Sequential Concrete* in a cognitive system, it creates a form that can be observed and interpreted in a structured way. Both algebraic thinking processes with types *Generational Representation of Concrete Random* able to meet all cognitive levels, S2 subjects solve algebraic problems as a whole by fulfilling algebraic thinking indicators. *Representation of Concrete Random* marked by students being able to use their previous knowledge in the form of language symbols and numerics, but in terms of representation they are not good where they are imperfect in interpreting information and relationships visually, numerical symbols verbally, so that in mathematical modeling they do not represent complex situations using mathematical expressions to describe relationships from an activity. The thinking process of students in solving problems is studied based on the cognitive system of Marzano's taxonomy. Through the cognitive system, concepts can be constructed which are then revealed in written and oral assignments.

Suggestion

Many recommendations are based on research findings. To begin with, lecturers must always manage the process of forming students' algebraic thinking in order to reduce student errors in uncovering mathematical problems in algebraic thinking. More research is needed on cognitive processes in other math problems, such as trigonometry or geometry. In solving trigonometry or geometry problems, it requires the ability to connect applicable formulas and skills in using these formulas to solve everyday life problems related to trigonometry. Solving problems in trigonometry requires problem solving skills and algebraic thinking skills.

Limitation

This research is limited to 2 subjects with people with algebraic thinking skills based on the ability to solve mathematical problems which are tracked using Marzano's taxonomy based on cognitive systems. As a result, there is still a lot of room for learning algebraic thinking in the form of new questions with different topic features, self-efficacy, cognitive styles, and learning styles.

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